Unit IV

7. (a) Let F and E be fields, and let $\tau_1, \tau_2, ..., \tau_n$ be distinct embedding of F into E. Then prove that these are linearly independent over E.

(b) Prove that the group G (Q(α)/Q), where $\beta^5 > 1$ and $\beta \equiv 1$, is isomorphic to the cyclic group of order 4.

8. (a) Show that x^5 . $8x$, 6 is not soluable by radicals over Q.

(b) Prove that it is impossible to construct a cube with a volume equal to twice of the volume of a given cube by using ruler and compass only.

(c) Let E be a finite separable extension of a field F then prove that the following are equivalent :

(i) E is a normal extension of F.

(ii) F is the fixed field of G (E/F).

No. of Printed Pages : 04 **Roll No. ....**

AA-311

M. Sc. EXAMINATION, Dec. 2018

(First Semester)

(Re-appear Only)

MATHEMATICS

MAT-501-B

Algebra

Time : 3 *Hours*] [*Maximum Marks* : 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) State and prove Zassenhaus Lemma.

   (b) State Jordan-Hölder Theorem and using this prove Fundamental Theorem of Arithmetic.

2. (a) Define commutator subgroup. Let G be a group and $a,b,c \notin \mathrm{G}$. Then prove that :

$$[a, b^{-1}, c]^{b}\,[b, c^{-1}, a]^{c}\,[c, a^{-1}, b]^{2} > 1.$$

   (b) Find all composition series of the octic group and verify that they are equivalent.

Unit II

3. (a) Discuss the solubility of (i) D_n ($n \geq 3$), (ii) s_n ($n \geq 1$).

   (b) If H is a normal subgroup of G and if H and G/H are soluble, then prove that G is soluble. Conversely, if G is soluble, then prove H and G/H are soluble.

4. (a) Prove that the direct product of a finite number of nilpotent groups is Nilpotent.

   (b) Prove that a normal subgroup N $\equiv \{e\}$ of a nilpotent group G meets the centre of G non-trivially and contains [G, N] properly.

Unit III

5. (a) Prove that every finite extension is an algebraic extension but not conversely.

   (b) Determine the minimal polynomials over Q of (i) $\sqrt{2}$ + 5, (ii) $\sqrt{-1}$, $\sqrt{2}$.

6. (a) Let E be a normal extension of F and let K be a subfield of E containing F. Show that E is a normal extension over K. Give an example to show that K need not be a normal extension of F.

   (b) Construct fields with 8 and 9 elements.