(i) $\dfrac{u'(a)}{u(a)} \geq \dfrac{V'(a)}{V(a)}, u(a) \neq 0, V(a) \neq 0$

(ii) $u(a) = 0$, $V(a) = 0$.

Then show that $V(t)$ has at least as many zeros in $[a, b]$ as $u(t)$. **10**

(b) Solve the Ricati's equation :

$$x^2 y' + 2 - 2xy + x^2 y^2 = 0$$ **10**

Unit III

5. (a) Find non-trivial solutions of the SLBVP $\dfrac{d^2 y}{dt^2} + \lambda u = 0$, where $u(0) = 0$, $u(\pi) = 0$. **10**

(b) Find the eigen values and eigen function of the SLBVP $\dfrac{d}{dx}\left(x\dfrac{dy}{dx}\right) + \dfrac{\lambda}{x} y = 0$, $y'(1) = 0$, $y'\left(e^{2\pi}\right) = 0$. Where we assume that the parameter λ is non-negative. **10**

AA314

M.Sc. EXAMINATION, May 2019

(First Semester)

(B. Scheme) (Re-appear)

MATHEMATICS

MAT507B

Ordinary Differential Equations-I

Time : 3 *Hours*] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Define equicontinuous family of functions. State and prove Ascoli theorem. **10**

(b) Solve the I.V.P. $\frac{dy}{dx} = 2y - 2x^2 - 3$, $y(0) = 2$ by using Picard method up to third approximation. **10**

2. (a) If $f \in$ C(C$_1$L$_i$p$_{in}$D). Let φ_1 and φ_2 be $\in_1$ and $\in_2$ approximation solution of the equation $\frac{dy}{dx} = f(x, y)$ in D some interval (a, b). If $\in = \in_1 + \in_2$, then for all $t \in (a, b)$ show that :

$$|\varphi_1(t) - \varphi_2(t)| \le \delta e^{K(t-t_0)} + \frac{\in}{K}\left(e^{K(t-t_0)} - 1\right)$$

where K is Lip constant and δ is given as :

$|\varphi_1(t_0) - \varphi_2(t_0)| \le \delta, t_0 \in (a, b)$. **15**

(b) Write a short note on continuation of solutions. **5**

Unit II

3. (a) Define total differential equation. State and prove the necessary and sufficient conditions for the integrability of $Pdx + Qdy + Rdz = 0$. **15**

(b) Let $u_1(t)$ and $u_2(t)$ be non-trivial L.D. solution of the Homo L.D.E. on $[a, b]$ and $p(t) > 0$, then the zeros of $u_1(t)$ and $u_2(t)$ are identifically same. **5**

4. (a) Consider Homogeneous diff. equations :

$$\frac{d}{dt}\left(d(t)\frac{dy}{dt}\right) + q_1(t)u = 0,$$

where $p(t) > 0$ and

$$\frac{d}{dt}\left(p(t)\frac{dV}{dt}\right) + q_2(t)V = 0,$$

$q_1(t) > q_1(t)$ on $[a, b]$

if :

8. (a) Factories the operator on the L.H.S. of $(x+2)D^2y-(2x+5)Dy+2y=(x+1)e^x$ and hence solve it where $D=\frac{d}{dx}$. **10**

(b) Reduce the equation :

$$\frac{d^2y}{dx^2}-4x\frac{dy}{dx}+\left(4x^2-Dy\right)=-3e^{x^2}\sin 2x$$

to normal form and hence solve it. **10**

6. (a) Find the solution of Laplace equation in (two Dim). by using separation of variable method w.r.t. boundry condition. **10**

(b) Define SLBVP problem and show that the eigen values of SLBV problem are always real. **10**

Unit IV

7. (a) Solve the linear diff. equation :

$$x^2\frac{d^2y}{dx^2}-\left(x^2+2x\right)\frac{dy}{dx}+(x+2)y=x^3e^x$$

10

(b) Solve the differential equation $\frac{d^2y}{dx^2}+n^2y=\sec nx$ by using method of variation parameters. **10**