4. (a) Using generating function approach establish the expression for Legendre polynomial. **10**

(b) Prove that :

(i) $xL'_n(x) = nL_n(x) - nL_{n-1}(x)$

(ii) $H'_n(x) = 2nH_{n-1}(x)$. **10**

Unit III

5. (a) State and prove Cauchy integral formula. **10**

(b) Determine the analytic function whose real part is : **10**

$$x^3 - 3xy^2 + 3x^2 - 3y^2 + 1$$

6. (a) Determine the poles and residue at its poles of the function : **10**

$$f(z) = \frac{ze^{iz}}{z^4 + a^4}$$

(b) Using method of contour integration evaluate. **10**

$$I = \int_0^\infty \frac{x^2 dx}{\left(x^2 + 9\right)\left(x^2 + 4\right)^2}$$

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M. Sc. EXAMINATION, Dec. 2017

(First Semester)

(Main & Re-appear)

PHYSICS

PHY-501-B

Mathematical Physics

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Explain eigen values and eigen vector of a matrix. Find the eigen values and a set of mutually othogonal eigen vectors of : **10**

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

(b) Show that velocity and acceleration are contravarient vectors and gradient of a scalar are component of covariant vectors. **5**

(c) If $a_{\alpha\beta}x^{\alpha}x^{\beta} = 0$ then show that : **5**

$$a_{ij} + a_{ji} = 0$$

2. (a) Prove that eigen values of a skew Hermition matrix are either zero or purely imaginary. **4**

(b) Prove that sum of eigen values is equal to the trace of corresponding matrix. **6**

(c) Define matric tensor. Find the component of metric tensor in cylinderical coordinate system. **10**

Unit II

3. (a) Drive the relation : **4**

$$J_n(x) = (-2)^n x^n \left\{ \frac{d^n}{d(x^2)^n} \right\} J_o x$$

(b) Prove that **4**

$$\int_0^{\pi/2} J_1(x\cos\theta)\,d\theta = \frac{1-\cos x}{x}$$

(c) Solve the equation by Frobenius method : **12**

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$$

(c) Find : **4**

$$L\left[\frac{\sin 6t}{t}\right]$$

$$L\left[t^3 \sin te^{-2t}\right]$$

Unit IV

7. (a) Using Fourier series show that : **5**

$$\sum_{n=1}^{\infty}(-1)^{n+1}\frac{\sin nx}{x} = \frac{x}{2}$$

(b) State and prove convolution theorem for Fourier transform. Comment on the application of these relations in context of Physics. **8**

(c) Show that : **7**

$$L\left\{\int_0^{\infty}\frac{\sin(xt)}{\sqrt{x}}dx\right\} = \frac{\pi}{\sqrt{2s}}$$

8. (a) Define Dirac delta function and find Fourier tranform of δ function. **6**

(b) Solve the following differential equation using Laplace Transform :

$$x\frac{d^2y}{dx^2} + \frac{dy}{dx} + xy = 0$$

given $y(0) = 2$, $y'(0) = 0$. **10**