## Unit III

5. (a) Evaluate the integral using Cauchy integral theorem $\oint_{\mathrm{C}} \frac{d z}{z}$, where C is a simple closed curve.

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(b) Define an analytic function in a complex Domain D. State and prove necessary and sufficient conditions for a function to be analytic.
6. (a) Using the method of contour integration evaluate :

8

$$
\mathrm{I}=\int_{0}^{2 \pi} \frac{d \theta}{(5+4 \cos \theta)}
$$

(b) Use calculus of residue to evaluate : $\mathbf{8}$

$$
\int_{-\infty}^{\infty} \frac{d x}{\left(x^{2}+9\right)}
$$ 8

$\qquad$

## Unit I

1. (a) Determine the eigen values and corresponding eigen vectors of matrix :

$$
A=\left[\begin{array}{ccc}
1 & 0 & -1 \\
1 & 2 & 1 \\
2 & 2 & 3
\end{array}\right]
$$

(b) Explain and prove Quotient law of tensors. Under what conditions it is inapplicable. 10
2. (a) For a Hermitian matrix, prove that its eigen values are real and its eigen vectors are orthogonal.

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(b) If a contravariant tensor of rank two is skew-symmetric in one coordinate system, show that it is skewsymmetric in any coordinate system.

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(c) Explain the following :
(i) Metric Tensors
(ii) Contraction theorem for tensors.

## Unit II

3. (a) Solve the following differential equation using Frobenius method :

$$
\left(x^{2}-x\right) \frac{d^{2} y}{d x^{2}}-(1+3 x) \frac{d y}{d x}-y=0
$$

(b) Prove that:

$$
\begin{aligned}
& \mathrm{P}_{n}(1)=1, \mathrm{P}_{n}(-x)=(-1)^{n} \mathrm{P}_{n}(x), \\
& \mathrm{P}_{n}(-1)=(-1)^{n} \text { and } \\
& \mathrm{P}_{2 n+1}(0)=0
\end{aligned}
$$

4. (a) Establish Rodrigue formula for Legendre polynomials.

10
(b) Obtain an expression for Hermite polynomial of order $n$ using generating function approach.

6
(c) Prove that:

$$
\mathrm{J}_{\frac{1}{2}}(x)=\sqrt{\frac{2}{\pi x}} \cdot \sin x
$$

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(ii)

(iii) $f(t)=t^{2} \sin ^{3} 6 t$.
(b) Find Fourier transform of $\delta$ function. 4
(c) Find Fourier sine transform of function $e^{-|t|}$ and hence evaluate :

$$
\int_{0}^{\infty} \frac{t \cdot \sin m t}{\left(t^{2}+1\right)} d t
$$

(c) Prove that the function given by $u=x^{2}-y^{2}-2 x y-2 x+3 y+6$ is harmonic. 4

## Unit IV

7. (a) Find inverse Laplace transform of the following function by convolution theorem :

$$
\mathrm{F}(s)=\frac{s}{\left(s^{2}+a^{2}\right)(s+b)}
$$

(b) Find the even series expansion of $f(x)=r-x$ in the interval $0<x<\pi$.

10
8. (a) Find Laplace Transform of the following function :

10
(i)


5
P.T.O.

