(b) Find the bilinear transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w_1 = 1$, $w_2 = i$, $w_3 = -1$, respectively.

Unit IV

7. (a) What is the difference between Taylor's series and Laurent's series. Expand the function $f(z) = \dfrac{z^2 - 1}{(z+2)(z+3)}$ using Taylor's and Laurent's series in the region :

(i) $|z| < 2$

(ii) $2 < |z| < 3$

(iii) $|z| > 3$ **12**

(b) Discuss the following :

(i) Removable singularity

(ii) Essential singularity

(iii) Limit point of poles is a non-isolated essential singularity

BB-315

M. Sc. EXAMINATION, May 2017

(Second Semester)

(Main & Re-appear)

(MATH)

MAT-510-B

COMPLEX ANALYSIS

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) State necessary and sufficient conditions for a function $f(z)$ to be analytical.

Obtain necessary condition for a function to be analytical. **10**

(b) Show that function $f(z) = e^{-z^{-4}} \left(z \neq 0\right)$ and $f(0) = 0$ is not analytical at $z = 0$ although CR equations are satisfied at that point. **10**

2. (a) Find the radii of convergence of the following power series :

(i) $\sum \frac{2^{-n} z^n}{1 + in^2}$

(ii) $\sum_{n=0}^{\infty} \frac{n^2 \left(z^2 + 1\right)^n}{\left(1 + i\right)^n}$

Also find the domain of convergence. **10**

(b) State and prove Cauchy Hadmard theorem. **10**

Unit II

3. State and prove Cauchy theorem and Cauchy-Goursat theorem. State basic difference between these two theorems. **20**

4. (a) State and prove Liouville's theorem. **10**

(b) State Argument principle, Maximum modulus principle and Rouche's theorem. Using Rouche's theorem prove that all the roots of $z^7 - 5z^3 + 12 = 0$ lies between the circle $|z| = 1$ and $|z| = 2$. **10**

Unit III

5. Obtain necessary and sufficient conditions for a function $f(z)$ to be conformal mapping. **20**

6. (a) Define Mobius transformation, critical point and under what condition a Mobius transformation is normal. Find the fixed point and normal form of the bilinear transformation : **14**

$$w = f(z) = \frac{3z - 4}{z - 1}.$$

(iv) State Riemann and Weierstrass theorem

(v) Identity theorem.

8. (a) State and prove Cauchy's residue theorem and using it evaluate : **10**

$$\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz,\ \ C:|z|=1.5$$

(b) State and prove Rouche's theorem and using it prove fundamental theorem on algebra. **10**

(iv) State Riemann and Weierstrass theorem

(v) Identity theorem.

8. (a) State and prove Cauchy's residue theorem and using it evaluate : **10**

$$\int_C \frac{e^{2z}}{(z-1)(z-2)}\,dz,\ \ C:|z|=1.5$$

(b) State and prove Rouche's theorem and using it prove fundamental theorem on algebra. **10**