(b) If the mapping W = $f(z)$ is conformal, then show that $f(z)$ is an analytic function.

8. (a) Discuss the branches and branch points of the function log z.

(b) Find the general homographic transformation which leaves the unit circle invarient.

Unit V

9. (i) Find the conjugate harmonic of :

$$x^3 - 3xy^2 - 5y.$$

(ii) Find the residue of $\frac{z^3}{z^2 - 1}$ at $z = \infty$.

(iii) What are mesomorphic functions ? Illustrate with example.

(iv) State Laurent's theorem.

(v) State and prove Cauchy's inequality.

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18BB1905

M. Sc. EXAMINATION, May 2019

(Second Semester)

(C. Scheme) (Main Only)

MATHEMATICS

MAT510C

Complex Analysis

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) State and prove sufficient condition for a function to be analytic.

   (b) Prove that the function $u = e^x \cos y$ is harmonic. Find the harmonic conjugate v and the analytic function $f(z) = u + iv$.

2. (a) Examine the behaviour of the power series $\sum_{n=1}^{\infty} \frac{z^{4n}}{4n+1}$ on the circle of convergence.

   (b) State and prove Taylor's theorem.

Unit II

3. (a) If a function $f(z)$ is analytic within and on a simple closed cantour C and 'a' is any point inside C. Then prove that :

$$f'(a) = \frac{1}{2\pi i} \int_C \frac{f(z)}{(z-a)^2}\, dz$$

   (b) State and prove Morera's theorem.

4. (a) Show that every polynomial of degree n has exactly n zeros.

   (b) State and prove minimum modulus theorem.

Unit III

5. (a) State and prove Cassorati-Weierstrass theorem.

   (b) Find the residues for $f(z) = \frac{\cot \pi z}{(z-a)^2}$ at its poles.

6. (a) Prove that :

$$\int_0^{\pi} \frac{\cos 2\theta}{1 + a^2 - 2a\cos\theta} = \frac{\pi a^2}{1-a^2} (a^2 < 1)$$

   (b) State and prove Mitlag Leffler's expansion theorem.

Unit IV

7. (a) Show that the transformation $w = \frac{1}{z}$ transforms circles into circles and lines into lines.