Unit III

6. (a) Show that the periodic solution and closed path of the planar autonomous system are closely related. **8**

(b) Determine the nature and stability of critical point of the system :

$$\frac{dx}{dt} = (y+1)^2 - \cos x$$

$$\frac{dy}{dt} = \sin(x+y)$$ **7**

7. (a) State and prove Bendixson's non-existence theorem for existence of a limit cycle. **7**

(b) State and prove the theorem concening the asymptotic stability of the critical point of the non-linear system. **8**

Unit IV

8. (a) Reduce the following IVP into an integral equation : **7**

$$\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0,\ y(0) = 1,\ y'(0) = 1.$$

18BB1904

M. Sc. EXAMINATION, May 2019

(Second Semester)

(C Scheme) (Main Only)

MATHEMATICS

MAT508C

ORDINARY DIFFERENTIAL EQUATIONS–II

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **1** is compulsory. All questions carry equal marks.

(Compulsory Question)

1. (a) Write a note on the fundamental matrix Q(t) for a periodic system. **4**

(b) Describe the following : **4**

(i) Autonomous system and the phase plane

(ii) Trajectory of the system.

(c) Define Liapunove function giving an example to it. **3**

(d) Define Integral equation and discuss its classifications. **4**

Unit I

2. (a) Show that the set of all solutions of LHS of nth order $\frac{dy}{dt} = \mathrm{A}(t)y, t \,\mathrm{C} - \mathrm{I}$ form an n-dimensional vector space over the complex field. **7**

(b) Define Non-homogeneous linear system and find the expression for its solution. **8**

3. (a) State and prove Liouville's formula for the solutions of LHS. **10**

(b) If $\lambda_1, \lambda_2 - \lambda_n$ are the Ch. roots of C and $r_1, r_2, \ldots.., r_n$ are the Ch. roots of eWR, then prove that def

$$\phi(t) = \lambda_1, \lambda_2 \ldots\ldots, \lambda_n$$
$$= \exp\,(r_1 + r_2 \ldots\ldots. + r_n)\ \mathrm{W}.$$ **5**

Unit II

4. (a) Define critical points of an autonomous system and discuss their classification with suitable explanation. **10**

(b) Determine the nature and stability of critical points of the system $\frac{dx}{dt} = x - y, \frac{dy}{dt} = x + 5y$. **5**

5. (a) Determine the nature and stability of critical points if the roots of characteristic equation, are real, unequal and opposite sign. **8**

(b) Define stability and asymptotic stability of autonomous system and prove related theorem. **7**

(b) Solve the integral equation : **8**

$$\phi(x) = 2x - \pi + 4 \int_0^{\pi/2} \sin^2 x \, \phi(t) \, dt .$$

9. Determine the eigen values and eigen function of the homogeneous integral equation $\phi(x) = \lambda \int_0^1 k(x,t)\, \phi(t)\, dt$ where,

$$k(x,t) = \begin{cases} -e^{-t} \sinh x, & 0 \le x \le t \\ -e^{-x} \sinh t, & t \le x \le 1 \end{cases} .$$ **15**

(b) Solve the integral equation : **8**

$$\phi(x) = 2x - \pi + 4 \int_0^{\pi/2} \sin^2 x \, \phi(t) \, dt .$$

9. Determine the eigen values and eigen function of the homogeneous integral equation $\phi(x) = \lambda \int_0^1 k(x,t)\, \phi(t)\, dt$ where,

$$k(x,t) = \begin{cases} -e^{-t} \sinh x, & 0 \le x \le t \\ -e^{-x} \sinh t, & t \le x \le 1 \end{cases} .$$ **15**