4. (a) Prove that the necessary and sufficient condition for the existence of an instantaneous code with word length $(n_1, n_2 ...... n_N)$ is a set of positive integer $[n_1, n_2 ...... n_N]$ exists iff $\sum_{i=1}^{N} D^{-n_i} \leq 1$.

where D = size of code alphabet. **15**

(b) Define properties of optical coding. **5**

Unit III

5. State and prove that fundamentals theorem for desired code. **20**

6. (a) State and prove Fano's Inequality. **10**

(b) Define decoding schemes and Ideal observer with example. **10**

Unit IV

7. (a) Write a short note on the applications of information theory. **10**

(b) Prove maximality property of entropy functions. **10**

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CC-319

M. Sc. EXAMINATION, May 2018

(Third Semester)

(Re-appear Only)

MATHEMATICS

MAT619B

Information Theory

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Define models for a communication system. **10**

(b) The joint probabilities for a transmitter as $(x_1, x_2, x_3, x_4, x_5)$ and a receiver alphabet (y_1, y_2, y_3, y_4) are given below :

$$\begin{array}{c} \\ x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{array} \begin{array}{c} \begin{matrix} y_1 & y_2 & y_3 & y_4 \end{matrix} \\ \begin{bmatrix} 0.15 & 0 & .10 & 0 \\ 0.10 & 0.20 & 0 & 0.10 \\ 0 & 0.05 & 0.05 & 0.05 \\ 0 & 0 & 0.05 & 0.05 \\ 0 & 0.05 & 0.05 & 0 \end{bmatrix} \end{array}$$

Determine the marginal, conditional and joint entropies for this channel. **10**

2. Define marginal joint and conditional entropy functions and establish relations among them. **20**

Unit II

3. (a) Construct a Huffman code for the below symbols : **10**

Symbol	Pro.
x_1	0.20
x_2	0.17
x_3	0.16
x_4	0.15
x_5	0.10
x_6	0.09
x_7	0.07
x_8	—

Also determine the average code word length.

(b) If $\overline{n}$ (Average code word length) of a uniquely decipherable code for the random variables X then prove that $\overline{n} \geq \frac{H(X)}{\log D}$ with equality iff $p_i = D^{-n_i} \;\; \forall \; i = 1, 2, ...m$. **10**

8. (a) State and prove branching property of entropy function with example. **10**

(b) Define Axiomatic characterization of Shannon entropy due to Shannon and Fadeev. **10**

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(b) Define Axiomatic characterization of Shannon entropy due to Shannon and Fadeev. **10**