 Roll No. ....

CC-312

M.Sc. EXAMINATION, May 2017

(Third Semester)

(Re-appear Only)

MATHEMATICS

MAT-603-B

Partial Differential Equations

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt any *Five* questions. All questions carry equal marks.

1. (a) Discuss Transport equation and initial value problem.

(b) State and prove the theorem on Mean-value formulas for Laplace's equation.

2. (a) State and prove Harnack's inequality.

(b) Discuss Green's function and the method of derivation of Green's function.

3. (a) State and prove the theorem on Dirichlet's principle based on energy methods.

(b) Suppose $u \in C_1^2(U_T)$ solve the heat equation in U_T. Then :

$$u \in C^{\infty}(U_T)$$

4. (a) Derive Kirchhoff's formula for the solution of the initial-value problem :

$$u_{tt} - \Delta u = 0 \quad \text{in } \mathbf{R}^n \times (0, \infty)$$

$$u = g, \; u_t = h \quad \text{on } \mathbf{R}^n \times \{t = 0\}$$

in three dimensions.

(b) Discuss cone of dependence and prove the theorem on finite propagation speed.

5. (a) Solve the problem :

$$\begin{cases} x_1 u_{x_2} - x_2 u_{x_1} = u & \text{in U} \\ u = g & \text{on T} \end{cases}$$

where U is the quadrant $\{x_1 > 0, x_2 > 0\}$ and $T = \{x_1 > 0, x_2 > 0\} \subseteq U$.

(b) State and prove the theorem on Local existence theorem for the solution of the PDE :

$$F(Du(x), u(x), x) = 0 \; (x \in V)$$

with the boundary condition :

$$u(x) = g(x) \; (x \in T \cap V)$$

6. (a) State and prove the Lemma on Lipschitz continuity.

(b) Discuss the Lipschitz continuous function as a weak solution of an initial-value problem and prove the theorem on uniqueness of weak solutions.

7. (a) Discuss the properties of Fourier transforms assuming $u, v \in L^2(\mathbf{R}^n)$

(b) Discuss the technique of utilizing a potential function for the conversion of a non-linear system of PDE into a single linear PDE.