

Unit III

No. of Printed Pages : 06

Roll No.

5. (a) Show that the metrix is a positive definite quadratic form in du, dv and calculate the first fundamental magnitude for the surface. **10**

$$r = [u \cos v, u \sin v, f(u)]$$

- (b) Show that the length of the perpendicular as far as terms of second order, on the tangent plane to a surface at the point (u, v) from a neighbouring point $(u + du, v + dv)$ is : **10**

$$\frac{1}{2} (Ldu^2 + 2Mdudv + Ndv^2)$$

6. (a) Show that the curve bisecting the angle between the parametric curve are given by : **10**

$$Edu^2 - Gdv^2 = 0$$

DD-313

M. Sc. EXAMINATION, Dec. 2017

(Fourth Semester)

(Re-appear Only)

MATHEMATICS

MAT-606-B

Differential Geometry

Time : 3 Hours]

[Maximum Marks : 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Find the unit vector along the tangent to given curve and derive the formula to find the arc length. **10**
- (b) For the curve $x = a(3t - t^3)$, $y = 3at^2$ and $z = a(3t + t^3)$ show that : **10**

$$K = \tau \frac{1}{3a(1+t^2)^2}$$

2. (a) Prove that the necessary and sufficient condition for a curve to be a helix is that its curvature and torsion are in constant ratio. **10**
- (b) Show that the tangent lines to the locus of center of spherical curvature of a curve are parallel to the binormal lines of the curve at the corresponding points. **10**

Unit II

3. (a) Define surface and parametric curves and find the equation of tangent plane and normal plane to the surface $z = x^2 + y^2$ at the point $(1, -1, 2)$. **10**
- (b) The normal at the point P of the ellipse :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

meets that co-ordinate planes in G_1, G_2, G_3 ; prove that the ratio $PG_1 : PG_2 : PG_3$ are constant. **10**

4. (a) Prove that the envelope of a family of surfaces touches each member of the family at all points of its characteristics. **10**
- (b) Find the envelop of family of planes :

$$3a^2x - 3ay + z = a^3$$

and show that its edge of regression is the curve of intersection of the surface $y^2 = 2x, yx = z$. **10**

8. (a) Define torsion of a geodesic and show that torsion of geodesic vanishes in a principal direction. **10**
- (b) Prove that at every point of a geodesic the principle normal is normal to the surface. **10**

- (b) Define the following : **10**
- (i) Normal Curvature
- (ii) Principal Curvature
- (iii) Line of Curvature
- (iv) Principal Direction
- (v) Minimal Surface.

Unit IV

7. (a) Prove that the necessary and sufficient condition for a curve $u = \text{constant}$ to be geodesic on the surface is : **10**

$$GG_1 + FG_2 - 2GF_2 = 0$$

- (b) Prove that the curves of the family $\frac{v^2}{u^2} = \text{constant}$ are geodesics on a surface with metric : **10**

$$v^2 du^2 - 2uv du dv + 2u^2 dv^2 \quad (u > 0, v > 0)$$