Section III

5. (a) Explain second fundamental form and second order magnitudes and what is the geometrical interpretation of the second fundamental form. **10**

(b) Obtain Weingarten equations and hence deduce the formula : **10**

$$H\vec{N}_1 \times \vec{N}_2 = \left(LM - M^2\right)\vec{N}$$

6. (a) Prove that the normals to any surface at consecutive points of one of its line of curvature intersect. Conversely, if the normals at two consecutive points of curve drawn on a surface intersect the curve is a line of curvature. **10**

(b) State and prove Euler's Theorem. **10**

Section IV

7. (a) Show that a necessary and sufficiency condition for a curve v = constant to be geodesics on the general surface is $EE_2 + FE_1 - 2EF_1 = 0$ and also find the condition for the curve $u = c$ to be a geodesics. **10**

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DD-313

M. Sc. EXAMINATION, May 2018

(Fourth Semester)

(Main & Re-appear)

MATHEMATICS

MAT606B

Differential Geometry

Time : 3 *Hours*] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : The question paper will consists of four Sections. Each Section contains two questions and students are required to attempt *Five* questions in all, selecting at least *one* question from each Section. All questions carry equal marks.

Section I

1. (a) Define space curve and find the unit vector along the tangent to a given curve. Also derive tthe formula for arc length. **10**

   (b) What is Torsion ? Find an expression for the torsion at a point P of a given curve. Also show that the necessary and sufficient condition that a given curve is a plane curve is that $\tau = 0$ at all points. **10**

2. (a) Prove that in order the principal normals of a curve be binormals of another, the relation $a(K^2 + \tau^2) = bK$ must hold, where a and b are constants. **10**

   (b) Obtain the curvature and torsion of a curve given by the intersection of two surface. **10**

Section II

3. (a) Define Guassion form of surface. Prove that at points common to the surface $a(yz + zx + xy) = xyz$ and a sphere whose centre is the origin, the tangent plane to the surface makes intercepts on the axes whose sum is constant. **10**

   (b) Define family of surface and prove that any tangent to the surface $a(x^2 + y^2) + xyz = 0$ meets it again in a conic whose projection on the plane of xy is a rectangular hyperbola. **10**

4. (a) What is edge of regression and show that each characteristics touches the edge of regression ? **10**

   (b) Find the envelope of family of planes $3a^2x - 3ay + z = a^3$ and show that its edge of regression is the curve of intersection of the surfaces $y^2 = zx$, $xy = z$. **10**

(b) Derive canonical Geodesics equations and show that the curves $u + v$ = constant are geodesics on a surfaces with metric $\left(1+u^2\right)du^2 - 2uv\,du\,dv + \left(1+v^2\right)dv^2$. **10**

8. (a) Find the geodesic curvature of the parametric curve $v = c$ and also prove that all straight lines on a surface are geodesics. **10**

(b) Define Torsion of a Geodesic and derive the expression for torsion of a geodesic in term of principal curvature and if K and τ are the curvature and torsion of a geodesic, prove that $\tau^2 = (K - K_a)(K_b - K)$. **10**

(b) Derive canonical Geodesics equations and show that the curves $u + v$ = constant are geodesics on a surfaces with metric $\left(1+u^2\right)du^2 - 2uv\,du\,dv + \left(1+v^2\right)dv^2$. **10**

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(b) Define Torsion of a Geodesic and derive the expression for torsion of a geodesic in term of principal curvature and if K and τ are the curvature and torsion of a geodesic, prove that $\tau^2 = (K - K_a)(K_b - K)$. **10**