Unit IV

7. (a) State and prove Cauchy's Goursat theorem. **10**

(b) Evaluate $\int_C \frac{e^z}{z-2} dz$, C : $|z - 0| = 3$ with the help of Cauchy residue theorem. **5**

8. (a) State and prove Schwartz Reflection principle. **10**

(b) Expand $f(z) = \frac{1}{z(z-2)}$ in a Laurent series valid for :

(i) $0 < |z| < 2$

(ii) $|z| > 2$. **5**

HH-344

Dual Degree B. Sc.(Hons.)/M.Sc.

EXAMINATION, May 2018

(Eighth Semester)

(Main & Re-appear)

MATHEMATICS

MAT518H

Complex Analysis

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Discuss spherical representation of complex number with the help of Riemann Sphere. **7**

(b) Derive Lagrange's identity for complex numbers. **8**

2. (a) Define power series and discuss redius of convergence and circle of convergence. State and prove Abel's Limit Theorem. **10**

(b) Find the radius of convergence of the series : **5**

$$\frac{z}{2}+\frac{1.3}{2.5}z^2+\frac{1.3.5}{2.5.8}z^3+\text{.....}$$

Unit II

3. (a) Prove that a set is compact iff it is complete and totally bounded. **7**

(b) Prove that set of all bilinear transformations form a linear group w.r.t. product of transformations. **8**

4. (a) Define cross ratio. Show that cross ratio remains invariant under a bilinear transformation. **7**

(b) State and prove Lioville's theorem. **8**

Unit III

5. (a) Define complex integration. Derive Cauchy integral formula and extend it to multiply connected region. **8**

(b) Evaluate : **7**

$$\int_C \frac{dz}{z(z+\pi i)},$$

C : $|z + 3i| = I$

6. (a) State and prove maximum modulus principle. **8**

(b) Expand $f(z)=\frac{z}{z^4+9}$ as a Taylor series about $z = 0$. Find also radius of convergence. **7**