8. (a) If $T \in A(V)$, set of all linear transformation for V to V, has all its characteristic root in *f*, then there is a basis of V in which matrix representation T is triangular. **12**

(b) Define the following :

(i) Nilpotent transformation

(ii) Index and Nilpotency

(iii) Rational Canonical forms. **3**

HH-342

M. Sc. EXAMINATION, May 2017

(5 Years Integrated)

(Eighth Semester)

(Main & Re-appear)

MATHEMATICS

MAT-514-H

Rings and Modules

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Define the following :
   - (i) Cyclic modules
   - (ii) Simple and semi-simple modules
   - (iii) Free modules. **8**

   (b) State and prove Schur's Lemma. **7**

2. State and prove Fundamental structure theorem of finitely generated modules over principal ideal domain. **15**

Unit II

3. (a) Define the following :
   - (i) Neotherian and Artinian modules
   - (ii) Neotherian and Artinian rings
   - (iii) Nil and Nilpotent ideals in Neotherian ring. **8**

   (b) Every homomorphic image of a Neotherian module is Neoetherian. **7**

4. State and prove Hilbert Basis theorem. **15**

Unit III

5. State and prove Wedderburn-Artin theorem. **15**

6. Define the following :
   - (i) $\mathrm{Hom}_R(R, R)$
   - (ii) Opposite rings
   - (iii) Maschk's theorem (statement only)
   - (iv) Uniform modules
   - (v) Primary modules
   - (vi) Neother Laskar theorem. **15**

Unit IV

7. (a) Determine all possible Jordan Canonical forms for a linear operator :

   $T : V \to V$ whose characteristic polynomial is $\Delta(x) = (x-2)^3 (x-5)^2$. **12**

   (b) Define Jordan blocks and Jordan forms. **3**