No. of Printed Pages : 05 Roll No. ....

18C701

B. Sc. EXAMINATION, 2020

(Third Semester)

(Main & Re-appear)

(Maths)

DMT311B

ADVANCED CALCULUS

Time : 2½ Hours] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Four* questions in all. All questions carry equal marks.

1. (a) By defination, prove that :

$$f(x) = \begin{cases} x^2 \cos\dfrac{1}{x} & , \quad x \neq 0 \\ 0 & , \quad x = 0 \end{cases}$$

is continuous at $x = 0$.

(b) For $u = e^{xyz}$ find $\dfrac{\partial^3 u}{\partial x \partial y \partial z}$.

(c) Evaluate $\int\limits_0^{\pi} \log\left(1 + a\cos x\right) dx$, by using differentiation under integral sign.

(d) Evaluate $\int\limits_0^1 \int\limits_x^{\sqrt{x}} \left(x^2 + y^2\right) dxdy$.

2. (a) Prove that the function defined by $f(x) = \sin\dfrac{1}{x}$, $x \in \mathrm{R}^+$ is continuous but not uniformly continuous on R^+.

(b) Verify Rolle's theorem for the function $f(x) = (x-a)^m (x-h)^n$, in the interval $[a, h]$, where m and n are positive integers.

3. State and prove Darboux theorem.

4. (a) If $u = \tan^{-1} \dfrac{x^3 + y^3}{x - y}$, find the value of

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}.$$

(b) If $u = x^2 + y^2 + z^2$, $v = xy + yz + zx$, $w = x + y + z$, find $\dfrac{\partial(u, v, w)}{\partial(x, y, z)}$.

5. (a) Expand $e^x \sin y$ in powers of x and y as far as terms of third degree.

(b) If $z = 2u^2 - v^2 + 3w^2$, where $u = xe^y$, $v = ye^{-x}$, $w = \dfrac{y}{x}$ find $\dfrac{\partial z}{\partial x}$ and $\dfrac{\partial z}{\partial y}$.

6. (a) State and prove Young's theorem.

(b) Show that the function :

$$f(x,y)=\begin{cases}\dfrac{x^2y^2}{x^4+y^4}, & \text{for } (x, y)\neq(0, 0)\\ 0, & \text{for } (x, y)=(0, 0)\end{cases}$$

is not differentiable at the origin.

7. (a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube.

(b) Examine the function $f(x, y) = x^3 + y^3 - 63(x + y) + 12xy$ for maxima and minima.

8. (a) Change the order of integration in :

$$\int_0^1 \int_x^{\sqrt{2-x^2}} \left(\frac{x}{\sqrt{x^2+y^2}}\right) dx$$

and hence evaluate it.

(b) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

9. (a) Evaluate $\iiint\limits_{V} \frac{dxdydz}{x^2 + y^2 + z^2}$, where V is the volume of the sphere $x^2 + y^2 + z^2 = a^2$.

(b) Prove that relation :

$$\beta(m, n) = \frac{\Gamma(m)\Gamma(n)}{\Gamma(m+n)}.$$