No. of Printed Pages : 05 **Roll No. ....**

C512

B. Sc. EXAMINATION, 2020

(Third Semester)

(Main & Re-appear)

(Phy.)

DPH203

MATHEMATICAL PHYSICS

Time : 2½ Hours] [*Maximum Marks : 75*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Four* questions in all. All questions carry equal marks.

1. (a) Define a linear vector space. Describe dimension and basis of a linear vector space and give at least one example.

(b) If A and B are Hermition matrices, then show that i(AB-BA) is also Hermition.

(c) If H is a subgroup of G, then prove that there exist one to one correspondence between two left (or right cosets) of H in G.

(d) Prove that the function $u = x^3 - 3y^2x + 3x^2 - 3y^2 + 1$ satisfy Laplace equation and determine the corresponding regular function $u + iv$.

(e) Prove that $H_n(-x) = (-1)^n H_n(x)$.

2. (a) Obtained a set of four orthogonal vectors by Schmidt's method from the following vectors :

U = (1, 0, 0, 1) V = (1, 1, 0, 2)

W = (1, 1, 2, –3) X = (1, 1, 1, 1)

(b) Given that $\{\alpha, \beta, \gamma\}$ is a linearly independent set of vectors, show that the set $\{\alpha + \beta, \beta + \gamma, \alpha + \gamma\}$ is also linearly independent.

3. (a) Reduce to normal form the following matrix :

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5 & -10 \end{bmatrix}$$

(b) Find the adjoint and inverse of the matrix :

$$A = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

(c) Let the vector space V = R^3, show that W is a subspace of V where $W = \{(a,b,c) : a+b+c=0\}$.

4. (a) State and prove rearrangement theorem.

(b) What are reducible and irreducible representations of a group ? State properties of latter.

5. (a) Construct the character table for group of symmetric operations of a square.

   (b) Define cyclic group. Prove that the group of order 4 may or may not be a cyclic group.

6. (a) State and prove Cauchy integral theorem.

   (b) Using Cauchy integral formula, evaluate :

$$\int_C \frac{\left(5z^2 - 3z + 2\right)dz}{(z-1)^3}$$

   where C is a simple closed curve enclosing $z = 1$.

7. (a) State and prove Morera's theorem.

   (b) If $f(z)$ is analytic function of z, prove that :

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4\left|f'(z)^2\right|$$

8. (a) From the generating function approach establish the expression for Hermite polynomial.

(b) From the generating function approach deduce the expression :

(i) $xL_n'(x) = n\,L_n(x) - n\,L_{n-1}(x)$

(ii) $(2n + 1)\,P_n(x) = P_{n+1}'(x) - P_{n-1}'(x)$

(iii) $J_{-n}(x) = (-1)^n\,J_n(x)$.

9. (a) Solve the differential equation $xy'' + 3y' + 4x^3y = 0$.

(b) What is Wronskian ? Discuss the general method for obtaining the second linearly independent solution of second order differential equation for which Frobenius method yields only one solution.