No. of Printed Pages : 03 Roll No. .......................

# 18A701

## B. Sc. (Hons.)-M. Sc. Dual Degree EXAMINATION, 2021

(First Semester)

(B-Scheme) (Main & Re-appear)

MATHEMATICS

DMT211B

Algebra

*Time : 2½ Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

**Note** : Attempt *Four* questions in all. All questions carry equal marks.

**1.** (a) If A and B are symmetric matrices, prove that AB is symmetric if and only if AB = BA.

(b) Prove that if two vectors are linearly dependent, then one of them is the scalar multiple of the other.

(c) Prove that transpose of a unitary matrix is unitary.

(d) Solve the equation $4x^3+16x^2-9x-36=0$, the sum of two of the roots being zero.

(e) Show that the equation $x^{2n}-1=0$ has only one real root.

**2.** (a) Find non-singular matrices P and Q such that PAQ is in the normal form :

$$A=\begin{bmatrix}3 & 1 & 2 & 1\\ 1 & 4 & 6 & 1\\ 2 & -3 & 1 & -2\end{bmatrix}$$

(b) Show that every square matrix can be expressed in one and only one way as P + iQ where P and Q are Hermitian matrices.

**3.** (a) Use Cayley-Hamilton theorem to find $A^{-1}$, if :

$$A = \begin{bmatrix} 1 & 2 & -1 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{bmatrix}$$

(b) Express $A = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix}$ as the product of elementary matrices.

**4.** (a) Find the values of $a$ and $b$ for which the following system of linear equations :

$$2x + by - z = 3$$

$$5x + 7y + z = 7$$

$$ax + y + 3z = 0$$

has an infinite number of solutions.

(b) Find the value of $k$ such that the system of equations :

$$x + ky + 3z = 0$$

$$4x + 3y + kz = 0$$

$$2x + y + 2z = 0$$

has a non-trivial solution.

**5.** (a) If A is a real skew-symmetric matrix such that $A^2 + I = 0$, show that A is orthogonal and is of even order.

(b) Diagonalize the quadratic form :

$$4x^2 + 10y^2 + 11z^2 - 4xy + 12zx - 12yz = 0$$

6. (a) Solve the equation :

$$x^4 + 15x^3 + 70x^2 + 120x + 64 = 0$$

whose roots are given to be in G.P.

(b) Find the condition that the two roots of the equation $ax^3 + bx^2 + cx + d = 0$ be equal.

7. (a) Remove the second term forms the equation $x^4 + 4x^3 + 2x^2 - 4x - 2 = 0$ and solve it.

(b) If $\alpha, \beta, \gamma$ are the roots of the equation $x^3 + px^2 + qx + r = 0$; form an equation whose roots are $\frac{\beta+\gamma}{\alpha}, \frac{\gamma+\alpha}{\beta}, \frac{\alpha+\beta}{\gamma}$.

8. (a) Solve the equation :

$$x^3 - 12x + 8 = 0$$

by Cardan's method.

(b) Solve the equation :

$$x^4 - 2x^3 - 5x^2 + 10x - 3 = 0$$

by Descarte's method.

9. (a) Solve the equation :

$$x^4 - 4x^3 - 4x^2 - 24x + 15 = 0$$

by Ferrari's method.

(b) Show that the equation :

$$2x^7 + 3x^4 + 3x + k = 0$$

has at least four imaginary roots for all values of $k$.