

**A514****B. Sc. (Hons.)-M. Sc. Dual Degree EXAMINATION, 2021**

(First Semester)

(Main &amp; Re-appear)

PHY

DMT101

MATHEMATICS–I

Time : 2½ Hours]

[Maximum Marks : 75

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Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

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**Note :** Attempt *Four* questions in all. All questions carry equal marks.

1. (a) Find  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$ .

(b) If  $z = x^3 + y^3 - 3axy$ , find  $\frac{\partial z}{\partial x}$  and  $\frac{\partial z}{\partial y}$ .

(c) Evaluate  $\int_1^2 \int_1^3 xy^2 dx dy$ .

(d) Find the equation of the plane passing through the points (2, 2, -1), (3, 4, 2) and (7, 0, 6).

(e) Find the equation of circular cylinder whose guiding curve is  $x^2 + y^2 + z^2 = 9$ ,  $x - y + z = 3$ .

2. (a) Show that  $f(x, y) = \begin{cases} \frac{2xy}{x^2 + y^2}, & (x, y) \neq (0, 0) \\ 0, & (x, y) = (0, 0) \end{cases}$  is continuous at every point except the origin.
- (b) Describe the level surface of the function  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ .
3. (a) Find the value of  $\frac{\partial^2 f}{\partial x^2}$  and  $\frac{\partial^2 f}{\partial y^2}$  at the point  $(4, -5)$  if  $f(x, y) = x^2 + 3xy + y - 1$ .
- (b) Find the gradient of function  $f(x, y) = \frac{x^2}{4} + y^2$ .
4. (a) Calculate  $\iint_R f(x, y) dA$  for  $f(x, y) = 1 - 6x^2y$  and  $R: 0 \leq x \leq 2, -1 \leq y \leq 1$ .
- (b) Find volume enclosed by the given region  $\int_0^1 \int_0^2 \int_0^{1-z} dy dx dz$ .
5. (a) Evaluate the given integral  $\int_0^{2\pi} \int_0^1 \int_0^{\sqrt{2-r^2}} dz r dr d\theta$ .
- (b) Integrate  $f(x, y, z) = x - 3y^2 + z$  over the line segment C joining the origin and the pt.  $(1, 1, 1)$
6. (a) Find the new co-ordinates of the point  $(1, -2)$  if the origin is shifted to  $(1, -1)$ , the new axes being paralalled to the original.
- (b) Find the transformed equation of  $17x^2 - 16xy + 17y^2 - 225 = 0$ , where the axes are rotated through an angle of  $45^\circ$ .

7. (a) A particle is moving in  $x$ - $y$ -plane. If co-ordinates are  $(x, y) = (3t, 4t^2)$ , find  
 (i) position vector at any time  $t$  (ii) Find  $r$  and  $\theta$  at  $t = 1$  sec.  
 (b) Prove that the equation  $2x^2 + xy - 6y^2 + 7y - 2 = 0$  represents a straight line.
8. (a) Find the equation of sphere of centre is  $(2, -3, 4)$  and radius is 5.  
 (b) Find the equation of the cone with vertex  $(0, 0, 0)$  and passing through the circle  $x^2 + y^2 + z^2 + x - 2y + 3z - 4 = 0$  and  $x - y + z = 2$ .
9. (a) Find the equation of the tangent plane to the coincide  $ax^2 + by^2 + cz^2 = 1$  at the point  $(1, 2, 3)$ .  
 (b) Prove that the plane  $8x - 6y - z = 5$  touches the paraboloid  $\frac{x^2}{2} - \frac{y^2}{3} = z$  and find the co-ordinates of the point of contact.