CC342

Dual Degree-B.Sc. (Hons.) Mathematics-M. Sc. (Mathematics) EXAMINATION, 2020

(Third Semester)

(B. Scheme) (Re-appear)

(B. Sc. (Hons.) M. Sc. (Mathematics))

MAT313H

PARTIAL DIFFERENTIAL EQUATIONS

Time : 2½ Hours]   [*Maximum Marks : 75*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Four* questions in all. All questions carry equal marks.

1. (a)   Form the partial differential equation by eliminating the arbitrary constants :

   (i) $4z = \left(ax + \dfrac{y}{a} + b\right)^2$

   (ii)   Form PPE by eliminating arbitrary function :

   $$z = f(xy) + g\left(\dfrac{x}{y}\right)$$

   (b)   Solve the equation :

   $$\left(x^2 - y^2 - z^2\right)p + 2xyq = 2xz$$

2. (a) Find a complete integral of :

$$2(pq + yp + qx) + x^2 + y^2 = 0$$

by using Charpit method.

(b) Find the complete integrals of the equation by using Jacobi's method :

$$2p_1 x_1 x_3 + 3p_2 x_3^2 + p_2^2 p_3 = 0$$

3. (a) Solve the equation :

$$2r - s - 3t = \frac{5e^x}{e^y}$$

(b) Solve the equation :

$$\left(D^2 - D'^2 + D + 3D' - 2\right) z = e^{x-y} - x^2 y$$

4. (a) Solve the equation :

$$\left(D^2 - DD' - 2D\right) z = \sin(3x + 4y)$$

(b) Solve the equation :

$$\left(x^2 D^2 - 2xyDD' - 3y^2 D'^2 + xD - 3yD'\right) 2 = x^2 y \cos\left(\log x^2\right)$$

5. (a) Reduce the PDE to its canonical form and hence solve it :

$$x^2 \frac{\partial^2 z}{\partial x^2} - y^2 \frac{\partial^2 z}{\partial y^2} = 0$$

(b) Solve the equation :

$$r + 5s + 6t = 0$$

6. (a) Solve the equation :

$$r - t\cos^2 x + p\tan x = 0$$

(b) Classify and reduce the equation :

$$\frac{\partial^2 z}{\partial x^2} - 4\frac{\partial^2 z}{\partial x \partial y} + 4\frac{\partial^2 z}{\partial y^2} = 0$$

to canonical form and hence solve it.

7. Solve :

$$\frac{\partial^2 y}{\partial t^2} = c^2 \left(\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} \right)$$

subject to b.c. $u(0, y, t) = u(a, y, t) = u(x, 0, t) = u(x, b, t) = 0, 0 \le x \le a, 0 \le y \le b$ and initial conditions :

$$u(x, y, 0) = f(x, y) \text{ and } \left(\frac{\partial y}{\partial t} \right)_{t=0} = g(x, y)$$

8. (a) Derive one dimensional wave equation by using method of separation of variable.

(b) Find the real characteristics of the equation :

$$e^{2x} \frac{\partial^2 y}{\partial x^2} + 2e^{x+y} \frac{\partial^2 y}{\partial x \partial y} + e^{2y} \frac{\partial^2 y}{\partial y^2} = 0$$