D515

Dual Degree B. Sc. (Hons.) Physics–M. Sc. Physics
EXAMINATION, 2020

(Fourth Semester)

(Main & Re-appear)

MATHEMATICS–IV

DMT202

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **9** is compulsory. All questions carry equal marks.

Unit I

1. (a) Find Laplace transform of function $f(t)$ where :

$$f(t) = \begin{cases} \sin t, & \text{if } 0 < t < \pi \\ 0 \;, & \text{if } \pi < t < 2\pi \end{cases}$$

given the $f(t)$ has period 2π. **8**

(b) Solve the differential equation :

$$\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} - 3y = \sin t;$$

where $y(0) = 0$ and $y'(0) = 0$; by Laplace transform method. **7**

2. (a) Find the Fourier transform of $f(x)$ defined by :

$$f(x)=\begin{cases}1; & |x|<a\\ 0; & |x|>a\end{cases}$$

and here evaluate $\int_0^{\infty}\frac{\sin as}{s}\,ds$. **8**

(b) Find Fourier cosine transform of $f(x)=e^{-x^2}$. **7**

Unit II

3. (a) If :

$$\sin(\alpha+i\beta)=x+iy,$$

prove that $\dfrac{x^2}{\sin^2\alpha}-\dfrac{y^2}{\cos^2\beta}=1$. **8**

(b) Show that the function :

$$f(z)=\sqrt{|xy|}$$

is not analytic at origin, even through Cauchy-Riemann equations are satisfied there at. **7**

4. (a) If $f(z)$ is an analytic function with constant modulus, show that $f(z)$ is a constant function. **8**

(b) Prove that the function :

$$u(x, y)=y^3-3x^2y;$$

is a Harmonic function. Determine its Harmonic Conjugate. **7**

Unit III

5. (a) Write down all characteristics of standard form of an LPP (linear programming problem) and discuss with an example of an LPP with 'n' variables and 'm' constraints. **8**

(b) Solve the linear programming problem (LPP) : **7**

Minimize Z = $20x + 10y$

Subject to

$$x + 2y \leq 40$$
$$3x + y \geq 30$$
$$4x + 3y \geq 60$$
$$x, y \geq 0$$

6. (a) Graph the feasible region of the LPP : **8**

Minimize Z = $6x + 10y$

Subject to the constraints

$$x \geq 6$$
$$y \geq 2$$
$$2x + y \geq 10$$
$$x, y \geq 0$$

and identifying the redundant constraints and then find solution.

(b) Solve the LPP by simplex method : **7**

Maximize Z = $2x_1 + x_2 - x_3$

Subject to constraints

$$x_1 + x_2 \leq 1$$
$$x_1 - 2x_2 - x_3 \leq -2$$
$$x_1, x_2, x_3 \geq 0$$

Unit IV

7. (a) A random variable X has the following probability distribution :

Values of X → X	:	0	1	2	3	4	5	6	7	8
P(X)	:	K	3K	5K	7K	9K	11K	13K	15K	17K

Find :

(i) Value of K

(ii) P $(2 \leq X \leq 5)$

(iii) Mean of Distribution

(iv) Variance of Distribution. **8**

(b) A bag contains 4 white and 3 red balls. These balls are drawn, with replacement, from this bag. Find the probability distribution, its mean, variance and standard deviation for number of red balls drawn. **7**

8. (a) Find the expression for mean and variance of the Binomial distribution. **8**

(b) In a Normal distribution 31% of the items are under 45 and 8% are over 64. Find mean and standard deviation of the distribution. **7**

(Compulsory Question)

9. (a) State and prove Modulation property of Fourier Transformation. **3**

(b) Split into real and imaginary parts : **3**

(i) $e^{(5+3i)^2}$

(ii) $\log(\alpha + i\beta)$.

(c) If a random variable has Poisson distribution such that P(1) = P(2). Find the mean of distribution. **3**

(d) Graph the linear inequality $3x + 4y \leq 12$. **3**

(e) Prove that sin z is a periodic function with period 2π. **3**