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FF343

M. Sc. (5 Year Integrated) EXAMINATION, 2020

(Sixth Semester)

(B Scheme)

(Main & Re-appear)

MATHEMATICS

MAT416H

DYNAMICS

B. Sc. (Hons.) M. Sc. (Mathematics)

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Derive the expression for radial and transverse components of velocity of a particle moving along a plane curve $r = f(\theta)$.

(b) A particle describes an angular spiral $r = ae^{m\theta}$ with constant angular velocity. Find its velocity and acceleration.

2. (a) Prove that acceleration of a point moving in a curve with uniform speed is $\rho\left(\frac{\partial\psi}{dt}\right)^2$.

(b) A weight is attached to the lower end of a light spiral spring whose upper end is fixed and released. If it oscillates through a space of $\frac{1}{2}$ m, then what is the period of oscillation ?

Unit II

3. (a) A force of 150 newtons acts on a body of mass 15 kg for 5 minutes and then ceases. What is the force required to bring the body to rest in 2 minutes ?

(b) An engine of horse power H, draws a train M tons up an incline of 1 in *n* against a resistance of *m* lbs wt per ton. Show that maximum speed of train is $\frac{550\,Hn}{M\,(2240 + mn)}$ ft/sec.

4. (a) State and prove the principle of conservation of energy, moving under the conservation system of forces.

(b) Show that if a mass *m* is allowed to slide down a smooth inclined plane, the sum of potential and kinetic energies at every instant is the same.

Unit III

5. (a) A particle slides down the outside of a smooth verticle circle starting from rest at the highest point. Discuss the motion.

(b) A particle attached to a fixed peg O by a string of length *l* is lifted up with the string horizontal and then let go. Prove that when the string makes an angle θ with horizontal, the resultant acceleration is $g\sqrt{(1 + 3\sin^2\theta)}$.

6. (a) Two particles are let drop the cusp of a cycloid down the curve at an interval of time *t*. Prove that they will meet in time $2\pi\sqrt{\frac{a}{g}} + \frac{t}{2}$.

(b) Show that a given gun will shoot three times as high when elevated at an angle of 60º and when elevated at 30º, but will carry the same horizontal range.

Unit IV

7. (a) A particle moves in an ellipse under a force which is always directed towards its focus; find (i) the law of force (ii) the velocity at any point of its path.

(b) The greatest and least velocities of a certain planet in its orbit round the sun are 30 km/sec and 29.2 km/sec respectively. Find the eccentricity of the orbit.

8. (a) A heavy particle moves in a smooth sphere. Show that if the velocity be that due to the level of the centre, the reaction of the surface will vary at the depth below the centre.

(b) A smooth helix is placed with its axis vertical and a small bead slides down it under gravity. Show that it makes its first revolution from rest in time $2\sqrt{\frac{\pi a}{g \sin \alpha \cos \alpha}}$, where α is the angle of the helix.