 Roll No. ......................

# HH343

## M. Sc. EXAMINATION, 2020

(5 Year Integrated)

(Eighth Semester)

(B Scheme)

(Main & Re-appear)

MATHEMATICS

MAT516H

General Topology

B. Sc. (Hons.) M. Sc. (Mathematics)

*Time :* 3 *Hours*] [*Maximum Marks :* 75



**Note** : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

**Unit I**

**1.** (a) Prove that every metric space is normal.

(b) Prove that every completely normal space is normal and hence every $T_5$-space is a $T_4$-space.

**2.** (a) Let (X, T) be a topological space. If X is a Hausdorff space, then so is quotient space.

(b) State and prove Urysohn's lemma.

## Unit II

3. (a) Let (X, $T_1$) and (Y, $T_2$) be two topological spaces. Then the collection $\{G \times H : G \in T_1 \text{ and } H \in T_2\}$ is a base for some topology for X × Y.

   (b) The product space is connected $\Leftrightarrow$ each space is connected.

4. (a) State and prove Tychonoff product theorem.

   (b) The product space $X = \pi\{X_\lambda : \lambda \in \wedge\}$ is $T_1$ iff each coordinate space $X_\lambda$ is $T_1$.

## Unit III

5. (a) Let (X, T) be a topological space and $Y \subset X$. Then a point $x_0 \in X$ is an accumulation point of Y iff there exists a net in $Y - \{x_0\}$ converging to $x_0$.

   (b) A topological space (X, T) is Hausdorff iff every net in X can converge to at most one point.

6. (a) Let $\mathbf{A}$ be a non-empty family of subsets of a set X. Then there exists a filter on X containing $\mathbf{A}$ iff $\mathbf{A}$ has the F.I.P.

   (b) A topological space (X, T) is Hausdorff iff every convergent filter in X has a unique limit.

## Unit IV

7. (a) Let $\mathbf{A}$ be an open, locally finite cover of a normal space X. Then for each $A \in \mathbf{A}$, there exists an open set G(A) s.t. $\overline{G(A)} \subset A$ and the family $\{G(A) : A \in \mathbf{A})\}$ covers X.

   (b) Prove that every regular Lindeloff space is paracompact.

8. (a) State and prove Michaell theorem on characterisation of paracompactness in regular spaces.

   (b) State and prove Nagata-Smirnov metrization theorem.