Unit II

4. (a) If $u = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, then find the value of : **7½**

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$$

(b) If $u = xyz, v = x^2 + y^2 + z^2, w = x + y + z$, then find the Jacobian $\frac{\partial(x, y, z)}{\partial(u, v, w)}$. **7½**

5. (a) Expand sin xy in powers of $(x-1)$ and $\left(y - \frac{\pi}{2}\right)$ upto second degree terms. **7½**

(b) Find $\int_0^a \frac{\log(1+ax)}{1+x^2} dx$ and hence evaluate $\int_0^1 \frac{\log(1+x)}{1+x^2} dx$. **5+2½**

A-2

B. Tech. EXAMINATION, Dec. 2018

(First Semester)

(B. Scheme) (Main & Re-appear)

(Common with 2nd Sem.)

MATH101B

MATHEMATICS-I

Time : 3 Hours] [*Maximum Marks : 75*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Q. No. **1** is compulsory. Attempt other *four* questions, selecting *one* question from each Unit. Marks are indicated against each.

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1. (a) Expand $f(x) = 2 + x^2 - 3x^5 + 7x^6$ in powers of $(x-1)$. **3×5=15**

(b) If $u = x^2 + y^2 + z^2$ and $x = e^{2t}$, $y = e^{2t}\sin 3t$, $z = e^{2t}\sin 3t$, find $\frac{dy}{dt}$.

(c) Evaluate $\int_0^a \int_y^a \frac{x\,dx\,dy}{x^2 + y^2}$ by changing into polar coordinates.

(d) Find the angle between the tangents to the curve $\vec{r} = t^2 i + 2tj - t^3 k$ at the pts. $t = \pm 1$.

(e) If $u = 3x^2 y, v = xz^2 - 2y$, find $\nabla(\nabla u . \nabla v)$.

Unit I

2. (a) Test the convergence of the following series :

(i) $1 + \frac{1}{2}.\frac{x^2}{4} + \frac{1.3.5}{2.4.6}.\frac{x^4}{8} + \frac{1.3.5.7.9}{2.4.6.8.10}.\frac{x^6}{12} + ...\infty$

(ii) $\sum_{n=1}^{\infty} \frac{8\tan^{-1} n}{1 + n^2}$ **4+3½**

(b) State the Leibnitz test for the convergence of an alternative series and hence determine whether or not $\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{(3n-2)}$ is absolutely convergent ? **3½+4**

3. (a) Show that : **5**

$$e^{x\cos x} = 1 + x + \frac{x^2}{2} - \frac{x^3}{3} + ...$$

(b) Find the equation of the circle of curvature for the curve $x^3 + y^3 = 3xy$ at the pt. $\left(\frac{3}{2}, \frac{3}{2}\right)$. **5**

(c) Find all asymptotes of : **5**

$$x^3 - x^2 y - xy^2 + y^3 + 2x^2 - 4y^2 + 2xy + x + y + 1 = 0$$

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Unit IV

8. (a) Show that the angular velocity at any pt. is equal to half the curl of the linear velocity at that pt. of the body. **7½**

(b) Show that $\operatorname{div}\left(\operatorname{grad} r^n\right) = n(n+1)r^{n-2}$ where $\vec{r} = xi + yj + zk$. **7½**

9. (a) Find the circulation of $\vec{F}$ round the curve C, where $\vec{F} = e^x \sin y i + e^x \cos y j$ and C is the rectangle. Whose vertices are (0, 0), (1, 0), $\left(1, \frac{\pi}{2}\right)$ and $\left(0, \frac{\pi}{2}\right)$. **7½**

(b) Verify Gauss divergence theorem for $\vec{F} = 4xzi - y^2 j + yzk$, taken over the cube bounded by $x = 0, x = 1$; $y = 0, y = 1$; $z = 0, z = 1$. **7½**

Unit III

6. (a) Find the volume of the solid formed by the revolution of the area enclosed by the curve $xy^2 = a^2(a - x)$ and its asymptote about y-axis. **7½**

(b) Find the surface generated by the revolution of the loop of the curve $3ay^2 = x(x - a)^2$ about the x-axis. **7½**

7. (a) Evaluate $\iint\limits_R xy\, dxdy$, where R is the region inside the square $|x| + |y| = 1$. **5**

(b) Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the hyperboloid $x^2 + y^2 - z^2 = 1$. **5**

(c) Find the relationship between Beta and Gamma function and hence evaluate : **3+2**

$$\int_0^1 x^3 (1 - x)^{4/3}\, dx.$$

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