5. (a) Solve by the method of variation of parameters :

$$\frac{d^2 y}{dx^2} - 6\frac{dy}{dx} + 9y = \frac{e^{3x}}{x^2}$$

(b) A second's pendulum which gain 10 seconds per day at one place losses 10 seconds per day at another; compare the acceleration due to gravity at the two places.

Part C

6. (a) Find the Laplace transform of the function :

(i) $f(t) = |t-1| + |t+1|, t \geq 0$

(ii) $f(t) = [t]$

where [ ] stands for the greatest integer function.

(b) Apply convolution theorem to evaluate :

$$L^{-1} \frac{s^2}{(s^2+a^2)(s^2+b^2)}$$

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B. Tech. EXAMINATION, May 2017

(Second Semester)

(Old Scheme) (Re-appear Only)

(Common for All Branches)

MATHEMATICS-II

MATH-102

Time : 3 *Hours*] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Part. All questions carry equal marks.

Part A

1. (a) Reduce the matrix :

$$A = \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

to normal form. Hence find the rank of A.

(b) For what values of parameters λ and μ do the system of equations :

$$x + y + z = 6$$
$$x + 2y + 3z = 6$$
$$x + 2y + \lambda z = \mu$$

have (i) no solution (ii) unique solution (iii) more than one solution.

2. (a) Find the eigenvalues and eigenvectors of the matrix :

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

(b) Are the following vectors linearly dependent ? If so, find a relation between them. $x_1 = (2, -1, 3, 2)$, $x_2 = (1, 3, 4, 2)$, $x_3 = (3, -5, 2, 2)$.

Part B

3. (a) Solve :

$$\left(xy^3 + y\right)dx + 2\left(x^2y^2 + x + y^4\right)dy = 0$$

(b) Find the orthogonal trajectories of the family of confocal conics :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2 + \lambda} = 1$$

where λ is a parameter.

4. (a) Solve :

$$\frac{d^2y}{dx^2} - 4y = x \sinh x$$

(b) Solve :

$$\left(x^2D^2 - xD - 3\right)y = x^2\left(\log x\right)^2$$

7. (a) Solve the simultaneous equations :

$$\frac{dx}{dt} + 5x - 2y = t, \frac{dy}{dt} + 2x + y = 0$$

given that $x = y = 0$ when $t = 0$.

(b) Solve :

$$\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$$

8. (a) Solve :

$$\left(p^2 + q^2\right)y = 9z$$

(b) A rod of length l with insulated sides is initially at a uniform temperature u_0. Its ends are suddenly cooled to 0ºC and are kept at that temperature. Find the temperature function $u(x, t)$.

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