(b) Solve :

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x\partial y}+y^2\frac{\partial^2 z}{\partial y^2}+nz$$

$$=n\left(x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}\right)+x^2+y^2+x^3$$

Unit III

5. (a) Classify and reduce the equation $\frac{\partial^2 z}{\partial x^2}-x^2\frac{\partial^2 z}{\partial y^2}=0$ to canonical form.

(b) Reduce the partial differential equation $\frac{\partial^2 z}{\partial x^2}-4\frac{\partial^2 z}{\partial x\partial y}+13\frac{\partial^2 z}{\partial y^2}-9\frac{\partial z}{\partial y}=0$ to canonical form.

6. (a) Solve the differential equation $t-r\sec^4 y=2q\tan y$ using Monge's method.

CC-342

Dual Degree/B.Sc. (Hons.)

EXAMINATION, Dec. 2018

(Third Semester)

(Main & Re-appear)

MATHEMATICS

MAT313H

Partial Differential Equations

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Find the differential equations of all spheres of fixed radius having centre in *xy*-plane.

(b) Solve the following :

(i) $p \tan x + q \tan y = \tan z$

(ii) $\left(x^2 - yz\right)p + \left(y^2 - zx\right)q = z^2 - xy$

2. (a) Define compatible system of partial differential equations of order one. Show that the equation $f\left(x, y, p, q\right) = 0$, $g\left(x, y, p, q\right) = 0$ are compatible if :

$$\frac{\partial\left(f, g\right)}{\partial\left(x, p\right)} + \frac{\partial\left(f, g\right)}{\partial\left(y, q\right)} = 0$$

(b) Find the complete integral of the equation $z^2 = 1 + p^2 + q^2$ by using Charpit's method.

Unit II

3. (a) Solve :

$$\frac{\partial^3 z}{\partial x^3} + \frac{\partial^2 z}{\partial x^2}\frac{\partial z}{\partial y} - \frac{\partial z}{\partial x}\frac{\partial^2 z}{\partial x^2} - \frac{\partial^3 z}{\partial y^3} = e^x \cos 2y$$

(b) If F(D, D') is a homogeneous function of $D := \frac{\partial}{\partial x}$ and $D' := \frac{\partial}{\partial y}$ of degree n and $u = ax + by$, then prove that :

$$\frac{1}{F(D, D')} f^{(n)}\left(u\right) = \frac{1}{F\left(a, b\right)} f\left(u\right)$$

where $f^{(n)}$ stands for nth derivative of f with respect to u as a whole and $F(a, b) \neq 0$. Hence :

$$\frac{1}{F(D, D')} f\left(u\right) = \frac{1}{F\left(a, b\right)} \underbrace{\iint \ldots \int}_{n \text{ times}} f\left(u\right) du du \ldots\ldots du$$

4. (a) Solve :

$$\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial y^2} + \frac{\partial z}{\partial x} + 3\frac{\partial z}{\partial y} - 2z = e^{x-y} - x^2 y$$

8. (a) Find the solution of $\frac{\partial^2 u}{\partial x^2} = \frac{1}{c^2}\frac{\partial u}{\partial t}$, $0 \le x \le 1$, $t > 0$ for which :

$u(0, t) = u(1, t) = 0$ and

$u(x, 0) = k \sin 2\pi x$

(b) Find the solution of two-dimensional Laplace equation $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$ in the region $0 \le x \le a, 0 \le y \le b$, satisfying the conditions :

$u(0, y) = u(a, y) = u(x, b) = 0$ and $u(x, 0) = x(a - x)$ for $0 < x < a$.

(b) Find the solution of the differential equation $\frac{\partial^2 z}{\partial x \partial y} = \frac{1}{x + y}$ such that $z = 0$, $\frac{\partial z}{\partial x} = \frac{2y}{x + y}$ on $y = x$.

Unit IV

7. (a) Find the characteristic of :

$$xy\frac{\partial^2 u}{\partial x^2} - \left(x^2 - y^2\right)\frac{\partial^2 u}{\partial x \partial y} - xy\frac{\partial^2 u}{\partial y^2} + y\frac{\partial z}{\partial x} - x\frac{\partial u}{\partial y} - 2\left(x^2 - y^2\right) = 0$$

(b) Find the solution of the wave equation :

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{c^2}\frac{\partial^2 u}{\partial t^2}$$

using the method of separation of variables.