6. (a) Prove that R is complete.

(b) State and prove cantor intersection theorem.

Unit IV

7. (a) Prove that the image of a Cauchy sequence under a uniformaly continuous mapping is itself a cauchy sequence.

(b) Prove that continuous image of a compact set is compact.

8. (a) Show that R is connected.

(b) Prove that compact sets have Bolzano-Weierstrass property.

No. of Printed Pages : 04 **Roll No. ....**

EE-341

Dual Degree/B.Sc. (Hons.)

EXAMINATION, Dec. 2018

(Fifth Semester)

(Main & Re-appear)

MATHEMATICS

MAT411H

Real Analysis

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) State and prove the necessary and sufficient conditions for Riemann integrability.

(b) If f is monotonic on $[a, b]$, then $f \in R[a, b]$.

2. (a) If $f \in R[a, b]$, then $|f| \in R[a, b]$ and $\left|\int_a^b f\right| \le \int_a^b |f|$. Give an example of a bounded function f such that $|f| \in R$ but f is not.

(b) Taking $f(x) = x$, $g(x) = e^x$, verify the second mean value theorem in $[-1, 1]$.

Unit II

3. (a) State comparison test for convergence of an integral and hence test the convergence of the integral $\int_0^\infty \frac{\cos x}{1+x^2} dx$.

(b) State Dirichlet's test for convergence of an integral and hence test the convegence of the integral :

$$\int_a^\infty \frac{1}{\sqrt{x}} \sin x dx, \ a > 0.$$

4. (a) Evaluate the integral :

$$\int_0^\infty \frac{\tan^{-1}(ax)}{x(1+x^2)} dx.$$

(b) Evaluate :

$$\int_0^\infty e^{-x^2} dx$$

Unit III

5. (a) Define a metric space and show that R^n is a metric space.

(b) Prove that in a metric space, every *nbd* is an open set.