where $\sigma_n(x) = \frac{1}{n}\sum_{k=0}^{n-1} S_k(x)$ and S_n is the nth partial sum of the Fourier series for f. **9**

(b) Applying Parseval's identity to the function :

$$f(t) = t \quad (-\pi \le t \le \pi.)$$

show that : **6**

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots\ldots + \frac{1}{n^2} + \ldots\ldots = \frac{\pi^2}{6}$$

Unit III

5. (a) Explain the following terms :

(i) Extended couple plane

(ii) Riemann sphere

(iii) Stereographic projection. **5**

(b) State and prove the necessary and sufficient condition for a function $f(z)$ to be analytic. **10**

FF-341

Dual Degree B.Sc. (Hons.) EXAMINATION, May 2018

(Sixth Semester)

(Main & Re-appear)

(MATHEMATICS)

MAT412H

Real and Complex Analysis

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Differentiate between a function of complex variable and a function of real variable. Also give suitable examples. **4**

(b) Define branch point of a function. Also find the branch points of the function : **5**

$$f(z) = \log\left(z^2 + z - 2\right)$$

(c) Using the identity $e^{iz} = \cos z + i \sin z$, show that : **6**

$$e^{iz_1} e^{iz_2} = \cos z_1 \cos z_2 - \sin z_1 \sin z_2 + i\left(\sin z_1 \cos z_2 + \cos z_1 \sin z_2\right)$$

2. (a) Find all roots of the equation $\sin z = \cosh y$ by equating the real parts and then the imaginary parts. **6**

(b) Define the domain of definition for the function : **5**

$$f(z) = \frac{z}{z + \overline{z}}$$

(c) Find the polar form of the complex number $-5 + 5i$. **4**

Unit II

3. (a) Define Fourier series for a function $f \in \rho[-\pi, \pi]$. Explain different properties of Fourier coefficients. Also find the Fourier series for the function : **10**

$$f(x) = \frac{x^2}{4} \quad [-\pi \le x \le \pi]$$

(b) Suppose $f \in \rho[-\pi, \pi]$ and

$$f(x) \sim \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos kx + b_k \sin kx\right)$$

show that if f is even, then $b_1 = b_2 = \ldots\ldots\ldots\ldots = 0$ while if f is odd then $a_0 = a = a_2 = \ldots\ldots\ldots\ldots = 0$. **5**

4. (a) If $f \in \rho[-\pi, \pi]$ and

$$f \sim \frac{a_0}{2} + \sum_{k=1}^{\infty}\left(a_k \cos kx + b_k \sin kx\right), \; -\pi \le x \le \pi.$$

show that :

$$\sigma_n(x) = \frac{a_0}{2} + \sum_{k-1}^{n-1}\left(1 - \frac{k}{n}\right)(a_k \cos kx + b_k \sin kx)$$

for $n = 1, 2, \ldots\ldots;\; -\pi \le x \le \pi$.

8. (a) Prove that at each point z of a domain D where $f(z)$ is analytic and $f'(z) \neq 0$, the mapping $w = f(z)$ is conformal. **9**

(b) Prove that cross ratio remains invariant under a bilinear transformation. **6**

6. (a) Show that : **5**

$$\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} = y \frac{\partial^2}{\partial z \, \partial \overline{z}}$$

where $z = x + iy$.

(b) Show that a harmonic function satisfies the differential equation : **3**

$$\frac{\partial^2 u}{\partial z \, \partial \overline{z}} = 0$$

(c) Show that the function :

$$u = \sin x.\cosh y + 2\cos x.\sinh y + x^2 - y^2 + 4xy$$

is a harmonic function. Also determine the corresponding analytic function $f(z) = u + iv$. **7**

Unit IV

7. (a) Show that product of two bilinear transformation is a bilinear transformation. **7**

(b) Find the general homographic transformations which leaves the unit circle invariant. **8**