

**Unit III**

6. (a) State and prove Jacobi-Poisson Theorem.  
(b) A particle of mass  $m$  moves in a force field whose potential in spherical coordinates is  $V = -\frac{\mu \cos \theta}{r^2}$ . Write the Hamilton-Jacobi equation describing its motion. Also find its solution.
7. (a) Show that the following transformation  $Q = \log\left(\frac{1}{q} \sin p\right); P = q \cot p$  is canonical.  
(b) Show that Poisson's bracket is invariant under canonical transformation.

**Unit IV**

8. (a) Find the expression for attraction of a thin spherical shell at any point outside the shell.

No. of Printed Pages : 05

Roll No. ....

**18AA1903**

**M. Sc. EXAMINATION, May 2019**

(First Semester)

(C Scheme) (Re-appear)

MATHEMATICS

MAT505C

Mechanics

*Time : 3 Hours*]

*[Maximum Marks : 75*

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Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

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**Note :** Q. No. 1 is compulsory. Attempt *Four* more questions by selecting *one* question from each Unit I-IV. All questions carry equal marks.

### Compulsory Question

1. (a) State and prove perpendicular axes theorem.
- (b) Define Scleronomic and Rheonomic systems. Also define generalized potential.
- (c) Derive Hamilton's Canonical equations.
- (d) Define Poisson's bracket along with its two properties.
- (e) Obtain the potential at an external point due to a solid sphere of mass  $M$ .

5×3=15

### Unit I

2. (a) Define principal axes. Prove that, in general, there are three principal axes through a point of a rigid body.
- (b) A uniform solid rectangular block is of mass  $M$  and dimensions  $2a \times 2b \times 2c$ . Find the equation of the momental ellipsoid for a corner of the block, referred to the edges through  $O$  as co-ordinate axes.

3. (a) Define equimomental system. Derive the necessary and sufficient conditions for two systems to be equimomental.
- (b) A square of side "b" has particles of masses  $m$ ,  $2m$ ,  $3m$  and  $4m$  at its vertices. Find the principal moments of inertia and principal directions at the centre of the square.

### Unit II

4. (a) State and prove Lagrange's equation of second kind.
- (b) Show that for a holonomic dynamical system, the kinetic energy is a quadratic function of velocities.
5. (a) State and derive Jacobi's equations for a conservative system.
- (b) Write a short note on Poincare-Cartan Integral Invariant.

- (b) Find the potential at a point on the axis of a uniform circular disc of radius “ $a$ ” and mass  $M$ .
9. (a) Derive Poisson's equation for potential in a system of attracting matter.
- (b) Show that a family of right circular cones with a common axis and vertex is a possible family of equipotential surfaces and find the potential function.

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