(b) Obtain a relationship between Riccati equation and linear differential equation of second order. Solve the Riccati equation :

$$\frac{dy}{dx} = (1-x)y^2 + (2x-1)y - x,$$

Unit III

5. (a) State and prove Sturm's fundamental comparison theorem. Also provide a suitable example.

(b) Find the characteristic values and functions for the Sturm-Liouville problem :

$$\frac{d^2y}{dx^2} + \lambda y = 0;\ y(0) = 0,\ y(n) - y'(n) = 0$$

6. (a) State and prove Sturm-Liouville theorem concerning the orthogonality of characteristic functions.

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AA-314

M. Sc. EXAMINATION, May 2018

(First Semester)

(Re-appear Only)

MATHEMATICS

MAT507B

Ordinary Differential Equations-I

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Explain the concept of equicontinuous family of functions. State and prove Arzela-Alcoli theorem.

(b) State and prove Cauchy-Euler construction theorem for an approximate solution of an IVP.

2. (a) State Picard-Lindelof theorem. For the initial value problem :

$$x'(t) = x^2 + t^2, x(0) = 0$$

find the largest interval $|t| \le h$ on which Picard's theorem guarantees the existence and uniqueness of solution.

(b) Obtain Picard's approximations of the differential equation $x'(t) = t + x$ with the initial condition (i) $x_0(t) = e^t$ and (ii) $x_0(t) = \cos t$.

Unit II

3. (a) Prove that a Pfaffian differential equation in two variables always possesses an integrating factor. Find out the necessary and sufficient condition for a Pfaffian differential equation in three variables to be integrable.

(b) State and prove comparison theorem for differential inequalities in the strip $a \le t \le b$.

4. (a) Let f and g be two solution of :

$$\frac{d}{dt}\left[P(t)\frac{dx}{dt}\right] + Q(t)x = 0$$

such that f and g have a common zero on $a \le t \le b$. Then prove that f and g are linearly dependent on $a \le t \le b$. Also prove the converse part stating the restriction.

8. (a) Solve the differential equation :

$$x^2 \frac{d^2y}{dx^2} + x\frac{dy}{dx} - y = x^2$$

by the method of variation of parameters.

(b) Solve the differential equation :

$$x\frac{d}{dx}\left(x\frac{dy}{dx} - y\right) - 2x\frac{dy}{dx} + 2y + x^2 y = 0$$

by removing the first derivative.

(b) Use the method of separation of variables to solve the heat equation :

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial x}$$

where the boundary conditions are :

$$u(0, t) = 0, u(\mathrm{L}, t) = 0, t > 0$$

and the initial condition is :

$$u(x, 0) = f(x), \quad 0 \le x \le \mathrm{L}.$$

where $\mathrm{L} > 0$ and f is a specified function of x, $0 \le x \le \mathrm{L}$.

Unit IV

7. (a) Find complete solution of linear differential equation :

$$xy^{(1)} - y = (x-1)\left(y^{(2)} - x + 1\right),$$

in terms of known integral.

(b) Describe the method of solution of linear differential equation of second order by changing the independent variable. Hence, solve :

$$y^2 + (3\sin x - \cot x)y^{(1)} + 2y\sin^2 x = e^{-\cos x}\sin^2 x$$