(b) Show that : **10**

(i) $H_{n+1}(x) = 2x\, H_n(x) - 2n\, H_{n-1}(x)$

(ii) $(2n + 1 - x)\, L^1_n(x) = L_n(x) + (n + 1)\, L^1_{n,1}(x) + n L^1_{n.1}(x)$

(iii) $P_n(-1) = (-1)^n$

Unit III

5. (a) What do you understand by an analytic function ? State and prove necessary and sufficient conditions for a function of complex variable to be analytic. Also the analyticity of $h(z) > \frac{1}{z}$. **10**

(b) Applying calculus of residue, prove that :

$$I = \int_0^{2\theta} \frac{\sin^2\rho}{a\,,\ b\cos\rho}\, d\rho > \frac{2\theta}{b^2}\left[a\ .\ \sqrt{a^2\ .\ b^2}\right]$$

where $a > b > 0$. **10**

AA-281

M. Sc. EXAMINATION, Dec. 2018

(First Semester)

(Re-appear Only)

PHYSICS

PHY-501-B

Mathematical Physics

Time : 3 Hours] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Define an orthogonal matrix. Show that the matrix : **4**

$$A = \begin{bmatrix} \cos\rho & \sin\rho \\ .\ \sin\rho & \cos\rho \end{bmatrix}$$

is orthogonal.

(b) Construct a general unitary matrix of order 2. **4**

(c) Prove that any two eigen vectors corresponding to two distinct eigen values of a Hermition matrix are orthogonal. **4**

(d) Find the mutually perpendicular eigen vectors of the matrix : **8**

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}.$$

2. (a) Prove that every tensor of second rank can be resolved into symmetric and antisymmetric tensor. **4**

(b) Define the fundamental tensor. Determine the fundamental tensor in terms of cylindrical coordinate system. **10**

(c) Component of a first rank tensor in rectangular Cartesion co-ordinate system are given by $xy, 2y\ .\ z^2, xz\ .$ Write its covariant component in spherical coordinates. **6**

Unit II

3. (a) Solve the following differential equation by Frobenius Method : **10**

$$xy'' , y' , x^2 y > 0 .$$

(b) Deduce the integral representation of a Bersel function. **6**

(c) Show that $Jn(.\ x) > (.\ 1)^n\ Jn(x)$. **4**

4. (a) Establish the expression for $P_n(x)$ from generating function technique. **10**

6. (a) Evaluate the following complex integration using Cauchy integral formula :

$$\int_{C} \frac{(3z^2 , z , 1)\, dz}{(z^2 . 1)(z , 3)}$$

where C is $|z| = 2$. **10**

(b) Prove that : **10**

$$\int_{0}^{f} \frac{\sin mx}{x}\, dx > \frac{\theta}{2}, m ? 0 .$$

Unit IV

7. (a) Find the Fourier series for the periodic function $f(x)$ given by :

$$f(x) > \begin{cases} . \theta & \text{if } . \theta = x = 0 \\ x & \text{if } 0 = x = \theta \end{cases}$$

Hence prove that

$$\frac{\theta^2}{8} > \frac{1}{1^2} , \frac{1}{3^2} , \frac{1}{5^2} , \text{........}$$ **7**

(b) Show that convolution of two Gaussian functions : **7**

$$F(t)\ e^{\cdot\ t^2/\beta^2} \text{ and } G(t) = e^{\cdot\ t^2/\chi^2}$$

is another Gaussian function

$$F(t) * G(t) = \beta\chi \left[\frac{\theta}{\beta^2 , \chi^2} \right] e^{\cdot\ t^2/(\beta^2 , \chi^2)}$$

(c) Find : **6**

(i) $L \left[t^2 e^{5t} \sin 3t \right]$

(ii) $L \left[\frac{\sin 6t}{t} \right]$

(iii) $L \left[\int_0^t \frac{\sin y}{t} dy \right]$.

8. (a) Evaluate : **6**

$$L^{-1} \left[\frac{1}{(s , 1)(s . 2)^2} \right]$$

(b) Show that : **8**

$$L \left[\int_0^f \frac{\sin tx}{\sqrt{x}} dx \right] > \frac{\theta}{\sqrt{2s}}.$$

(c) Find Fourier transform of function : **6**

$$f(t) > \begin{cases} 1 . t^2 & |t| / 1 \\ 0 & |t| ? 1 \end{cases}.$$