8. If μ is a measure defined on a σ-algebra M containing the Baire sets. Assume that either that μ is inner regular. Then for each $E \in M$ with $\sigma E < \infty$, there is a Baire set B with $\mu (E \Delta B) = 0$.

JJ341

M. Sc. EXAMINATION, May 2019

(5 Years Integrated)

(Tenth Semester)

(B Scheme) (Main & Re-appear)

B. Sc. (Hons.) M. Sc. (Mathematics)

MATHEMATICS

MAT612H

Inner Product Spaces and Advanced Measure Theory

Time : 3 Hours] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : The question paper consist of four Units. Each unit contains two questions and the students asked to attempt a total *five*, selecting atleast *one* question from each Unit.

Unit I

1. (a) Prove that Inner Product in an Hilbert space is jointly continuous.

   (b) Prove that in an inner product space, if $<x_n>$ and $<y_n>$ are Cauchy sequence of vectors, then $\{(<x_n, y_n>)\}$ is Cauchy sequence of scalars.

2. (a) Prove that a closed convex subset of a Hilbert Space H contains a unique vector of smallest norm.

   (b) State and prove riesz representation theorem for Hilbert Space.

Unit II

3. Prove that the adjoint operator $T \rightarrow T^*$ on B(H) has the following properties :

   (a) $(T_1 + T_2)^* = T_1^* + T_2^*$

   (b) $(\alpha T)^* = \overline{\alpha} T^*$

   (c) $(T_1 T_2)^* = T_2^* T_1^*$

   (d) $T^{**} = T$

   (e) $\|T^*\| = \|T\|$

4. (a) Prove that if T is an operator's on H, then T is normal $\Leftrightarrow$ its real and imaginary parts commute.

   (b) Prove that if P is the projection on a closed lienar subspace M of H, then M reduces an operator T $\Leftrightarrow$ TP = PT.

Unit III

5. Prove that if E and F are measurable sets and v is a signed measure such that $E \subset F$ and $|vF| < \infty$, then $|vE| < \infty$.

6. (a) State and prove Jordan Decomposition Theorem.

   (b) Prove that the set function $u \times v : S \times \Sigma \rightarrow [0, \infty]$ defined by $u \times v\,(A \times B) = u(A).v(B)$ for each $A \times B \in S \times \Sigma$ is a measure.

Unit IV

7. Prove that the class σ of μ-measurables set is a σ-algebra. if $\overline{\mu}$ is restricted to β, then $\overline{\mu}$ is a complete measure on β.