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BB315

M. Sc. EXAMINATION, 2020

(Second Semester)

(B Scheme)

(Re-appear)

MATHEMATICS

MAT510B

COMPLEX ANALYSIS

Time : 3 *Hours*] [*Maximum Marks* : 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) State necessary and sufficient condition for a function to be analytic. Prove the necessary condition for a function $f(z)$ to be analytic.

(b) Show that the function $u(x, y) = e^x \cos y$ is harmonic. Determine its harmonic conjugate and analytic function $f(z)$.

2. (a) Find the domain of convergence of the series :

(i) $\sum_{n=0}^{\infty} n^2 \left(\dfrac{z^2+1}{1+i}\right)^n$ (ii) $\sum_{n=1}^{\infty} 4^{-n} \dfrac{(z+2)^{n-1}}{(n+1)^3}$.

(b) State and prove Cauchy Hadmard Theorem.

Unit II

3. (a) Define Complex Integration. Verify Cauchy theorem for the function $f(z) = e^z$ along the boundary of triangle having vertices $1 + i$, $-1 + i$, $-1 - i$.

(b) State and prove Morera's Theorem.

4. (a) State and prove Maximum Modulus principle. Also deuce Minimum Modulus Principle.

(b) Evaluate the following integrals :

(i) $\int_c \frac{4-3z}{z(z-1)(z-2)} dz$, where $c : |z| = \frac{3}{2}$

(ii) Evaluate $\int_c |z| \, dz$, an taking c is upper half part and right half part.

Unit III

5. (a) Discuss the following transformations :

Jacobian, Translation, Rotation, Magnification and Inversion and Conformal.

(b) State necessary and sufficient condition for a transformation $w = f(z)$ to be conformal and prove necessary part only.

6. (a) Define the Mobius transformation. Find the Mobius transformation which maps the points $z_1 = 2$, $z_2 = i$, $z_3 = -2$ into the points $w_1 = 1$ $w_2 = i$, $w_3 = -i$.

(b) Define fixed point, cross ratio, inverse points and critical points. Find the fixed points and normal form of the Mobius transformation $w = \frac{3z-4}{z-1}$.

Unit IV

7. (a) State Taylor's and Laurent series. Express $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ as Taylor's or Laurent series in the regions $|z| < 2$, $2 < |z| < 3$.

(b) Discuss the nature of singularity of the following functions :

(i) e^z at $z = \infty$

(ii) $\dfrac{(z - \sin z)}{z^3}$ at $z = 0$

(iii) $f(z) = \sqrt{z-3}$ at $z = 3$.

8. (a) State Cauchy Residue theorem. Evaluate using Residue theorem $\int_0^\infty \frac{dx}{(1+x)^2}$.

(b) State Argument principle and Rouche's theorem. Using Rouche's theorem prove that all the roots of $z^8 - 5z + 14$ lies between the circles $|z| = 1$ and $|z|$ 2.