(b) Let a rectangular domain D be bounded by $x=0, y=0, x=2, y=1$. Determine the region $\mathrm{D}^{\prime}$ of the $w$-plane into which D is mapped under the transformation $w=z+(1-2 i)$.

10
6. (a) Define cross-ratios and show that they are invariant under a bilinear transformation.

10
(b) Find the fixed point and normal form of the bilinear transformation :

$$
\mathrm{W}=\frac{z-1}{z+1}
$$

## Unit IV

7. (a) Define Laurent's series. Expland the function $f(z)=\frac{1}{z^{2}-3 z+2}$ as a series in the region :
(i) $0<|z|<1$
(ii) $1<|z|<2$
(iii) $|z|>3$.

## BB-315

M. Sc. EXAMINATION, Dec. 2017
(Second Semester)
(Re-appear Only)
MATHEMATICS
MAT-510-B
Complex Analysis

Time : 3 Hours]
[Maximum Marks : 100
Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt Five questions in all, selecting at least one question from each Unit. All questions carry equal marks.
P.T.O.

## Unit I

1. (a) State and obtain sufficient condition for a function $f(z)$ to be analytical.

10
(b) If $w=f(z)=u+i v$ is analytical function and $u-v=e^{x}(\cos y-\sin y)$. Find W in terms of $z$.

10
2. (a) The sum function $f(z)$ of the series $\sum_{n=0}^{\infty} a_{n} z^{n}$ represent an analytical function inside the circle of convergence. 14
(b) Find the domain of convergence of the series :

$$
\sum_{n=1}^{\infty} \frac{1 \cdot 3 \cdot 5 \cdot \ldots \ldots .(2 n-1)}{n!}\left(\frac{1-z}{z}\right)^{h}
$$

## Unit II

3. (a) Define complex line integral and evaluate the integral :

10

$$
\int_{0}^{1+i}\left(x-y+i x^{2}\right) d z
$$

M-BB-315 2
(i) Along the straight from $z=0$ to $z=1+i$.
(ii) Along the real axis from $z=0$ to $z=1$ and then along a line parallel to the imaginary axis from $z=1$ to $z=1+i$.
(b) State and prove Cauchy's integral formula. 10
4. (a) State and prove Morera's theorem.

10
(b) State and prove maximum modulus principle and hence also deduce minimum modulus principle.

## Unit III

5. (a) State necessary conduction for $w=f(z)$ to be a conformal mapping and hence obtain it.
P.T.O.
(b) Discuss various kinds of singularity and prove that $f(z)$ can be explanded in form
$\frac{1}{z^{2}}-\frac{1}{2 z}+a_{0}+a_{1} z^{2}+a_{4} z^{4}+\ldots \ldots \ldots \ldots \ldots$.
where $0<|z|<2 \pi$ and $f(z)=\frac{1}{z\left(e^{z}-1\right)}$.
6. (a) State and prove Rouche's theorem and using this theorem prove that every polynomial of degree $n$ has $n$ zeros. 14
(b) Apply the calculus of residue, to prove that :

6

$$
\int_{0}^{\infty} \frac{d x}{1+x^{2}}=\frac{\pi}{2}
$$

(b) Discuss various kinds of singularity and prove that $f(z)$ can be explanded in form $\frac{1}{z^{2}}-\frac{1}{2 z}+a_{0}+a_{1} z^{2}+a_{4} z^{4}+$ $\qquad$ where $0<|z|<2 \pi$ and $f(z)=\frac{1}{z\left(e^{z}-1\right)}$.
8. (a) State and prove Rouche's theorem and using this theorem prove that every polynomial of degree $n$ has $n$ zeros. 14
(b) Apply the calculus of residue, to prove that : 6

$$
\int_{0}^{\infty} \frac{d x}{1+x^{2}}=\frac{\pi}{2}
$$

