(b) Define cross ratio. Prove that cross ratios are invariant under a bilinear transformation. **10**

## Unit IV

**7.** (a) If $f(z)$ is analytic in a circular domain D with centre $a$, then for every $z \in$ D :

$$f(z)=\sum_{n=0}^{\infty}\frac{(z-a)^n}{n!}f^n(a)$$

so that $f(z)$ can be expressed as a power series about $z = a$. **10**

(b) State Rouche's theorem and apply it to show that all roots of the equation $z^7 - 5z^3 + 12 = 0$ lies between the circles $|z| = 1$ and $|z| = 2$. **10**

**8.** (a) Discuss the nature of singularity of the following functions : **10**

(i) $f(z) = e^z$ at $z = \infty$





# BB315

## M. Sc. EXAMINATION, May 2019

(Second Semester)

(B. Scheme) (Re-appear)

MATHEMATICS

MAT510B

Complex Analysis

*Time : 3 Hours*] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

**Note** : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

## Unit I

1. (a) Obtain Cauchy-Riemann equation for an analytical function. **10**

   (b) Show that $f(z) - e^{-z^{4}} (z \neq 0)$ and $f(0) = 0$ is not analytic at origin although Cauchy-Riemann equations are satisfied at origin. **10**

2. (a) Show that function $u(x, y) = e^x \cos y$ is harmonic. Determine its conjugate and analytic function $f(z)$.

   (b) Find the domain of convergence of the series : **10**

$$\sum_{n=1}^{\infty} \frac{1.3.5......(2n-1)}{n!} \left( \frac{1-z}{z} \right)^n$$

## Unit II

3. (a) State and prove Cauchy's integral formula for higher order derivatives and evaluate the integral : **10**



$$\int_C \frac{e^z}{(z+1)^2} \, dz, \quad C : |z - 1| = 3$$

   (b) State and prove Morera's theorem. **10**

4. (a) State and prove Schwarz Lemma. **10**

   (b) State and prove Poisson's integral formula. **10**

## Unit III

5. (a) Define conformal mapping. State and prove sufficient condition for $w = f(z)$ to represent a conformal mappings. **10**

   (b) By the transformation $w = z^2$, show that the circle $| z - a | = c$, $a$ and $c$ being real, in $z$-plane correspond to Limacons : **10**

$$R = 2c(a + c\cos\phi)$$

6. (a) Prove that every bilinear transformation can be expressed as the resultant of an even number of inversions. **10**

(ii) $f(z) = \tan\frac{1}{2}$ at $z = 0$

(iii) $f(z) = \cos - \sin z$ at $z = \infty$

(iv) $f(z) = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{1+(2^n z)^2}$

(v) $f(z) = e^{-\frac{1}{z^2}}$

(b) Show by Contour integration : **10**

$$\int_0^{\infty} \frac{dx}{1+x^2} = \frac{\pi}{2}$$



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