5. (a) Find the equation of the cone with vertex (1, 1, 1) and passing through the curve of itner-section of $x^2 + y^2 + z^2 = 1$ and $x + y + z = 1$.

(b) Find the equation of the right circular cylinder whose axis is $x = 2y = -z$ and radius 4.

Unit III

6. (a) Prove that the six normals from a point to an ellipsoid lie on a curve of second degree.

(b) Find the equations of the tangent planes to $3x^2 + 2y^2 - 6z^2 = 5$ which pass through the lines $3x - y - 9z = 0$ and $6x + 3y - 3z - 5 = 0$.

7. (a) Find the locus of the straight lines drawn through a fixed point (α, β, γ) at right angles to its polars w.r.t the conicoid $ax^2 + by^2 + cz^2 = 1$.

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18B703

Dual Degree B. Sc. (Hons.) /M. Sc.

EXAMINATION, May 2019

(Second Semester)

(Main Only)

MATHEMATICS

DMT216B

SOLID GEOMETRY

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. Q. No. **1** is compulsory. Attempt all parts together of a question. All questions carry equal marks.

(Compulsory Question)

1. (a) Find the coordinates of the centre of the conic :

$$x^2 - 3xy + y^2 + 10x - 10y + 21 = 0 .$$

(b) Find the equation of the tangent to the conic :

$$x^2 + 2xy - y^2 + 2x + 4y + 1 = 0$$

at the point (– 2, 1).

(c) Find the centre and radius of the sphere :

$$2x^2 + 2y^2 + 2z^2 - 2x + 4y + 2z - 5 = 0 .$$

(d) Define Enveloping Cone.

(e) Define Elliptic Paraboloid.

Unit I

2. (a) Find the lengths, equations of exes and the eccentricity of the conic :

$$5x^2 - 24xy + 5y^2 + 14x + 8y - 16 = 0 .$$

(b) Find the equation of director circle of the conic :

$$14x^2 - 4xy + 11y^2 - 44x - 58y + 71 = 0 .$$

3. (a) Prove that in general two parabolas can be drawn through four given points.

(b) Prove that the conics :

$$x^2 - y^2 - 4x + 2y + 2 = 0$$

and

$$x^2 + 3y^2 - 4x - 6y + 4 = 0$$

are confocal.

Unit II

4. (a) Find the centres of the two spheres, which touch the plane $x + 2y + 2z - 5 = 0$ at the point (1, 1, 1) and the sphere $x^2 + y^2 + z^2 + 2x + 4y + 6z - 11 = 0 .$

(b) Two spheres of radii r_1 and r_2 cut orthogonally. Prove that the radius of the common circle is $\frac{r_1 r_2}{\sqrt{r_1^2 + r_2^2}}$.

(b) Find the locus of the centres of sections of the conicoid :

$$ax^2 + by^2 + cz^2 = 1,$$

which touch the conicoid

$$\alpha x^2 + \beta y^2 + \gamma z^2 = 1.$$

Unit IV

8. (a) Find the equation of the plane which cuts the paraboloid $x^2 - 2y^2 = z$ in a conic with its centre at the point

$$\left(2, \frac{3}{2}, 4\right).$$

(b) Find the lengths of semi-axis of the sections of the paraboloid

$$2x^2 + y^2 - z = 0$$

by the plane

$$x + 2y + z = 4.$$

9. (a) Prove that the surface whose equation is :

$$16x^2 + 4y^2 + 4z^2 - 4yz - 8zx + 8xy + 4x + 4y - 16z - 24 = 0$$

is an elliptic paraboloid.

(b) Show that the two confocal paraboloids cut everywhere at right angles.