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M. Sc. EXAMINATION, 2020

(Third Semester)

(C Scheme) (Main & Re-appear)

MATHEMATICS

MAT605C

ADVANCED COMPLEX ANALYSIS

Time : 2½ Hours] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Four* questions in all. All questions carry equal marks.

1. (a) State and prove Hurwitz's theorem.

(b) If z_1, z_2,......, Z_n,........ be any sequence of numbers whose only limit point is the point at infinity, then show that it is possible to construct an integral function which vanishes at each of the point z_n and nowhere else.

2. (a) Prove that :

$$\zeta(1-z) = 2^{1-z}\pi^{-z}\cos\frac{1}{2}\pi z\Gamma(z)\zeta(z).$$

(b) State and prove Runge theorem.

Unit II

3. (a) Show that the functions $1 + az + a^2z^2 + ...$ and

$$\frac{1}{1-z} - \frac{(1-a)^z}{(1-z)^2} + \frac{(1-a)^2 z^2}{(1-z)^3} + \ldots\ldots\ldots\ldots\ldots.$$

are analytic continuation of each other, where a is real.

(b) Describe power series method of analytic continuation.

4. (a) Let $\gamma : [0, 1] \to \mathbb{C}$ be a path from a to b and let $\{(f_t, D_t) : 0 \leq t \leq 1\}$ and $\{g_t, B_t) : 0 \leq t \leq 1\}$ be Analytic continuation along γ such that $[f_0]_a = [g_0]_a$. Prove that $[f_1]_b = [g_1]_b$.

(b) Outline the Dirichlet problem for a unit disc.

5. (a) State and prove Poisson Jensen's formula.

(b) If the real part of an entire function $g(z)$ satisfies the inequality Re $g(z) < r^{\rho+\varepsilon}$ for every $\varepsilon > 0$ and all sufficiently large r, then prove that $g(z)$ is a polynomial of degree not exceeding ρ.

6. (a) Show that if $f(z)$ is an entire function of order ρ and convergence exponent σ, then $\sigma \leq \rho$.

(b) Let $p(z)$ be a canonical product of finite order ρ and $q > 0$, $\varepsilon > 0$. Then show that for all sufficiently large $|z|$; $|z - z_i| > |z_i|^{-q}$ implies $\log|p(z)| > -|z|^{\rho+\epsilon}$.

7. (a) Let $f(z) = \{z : |z| < 1\}$ such that $f(0) = 0$, $f'(0) = 1$ and $|f(z)| \leq M$ for all z in D. Prove that $M \geq 1$ and $f(D) \supseteq B\left(0; \dfrac{1}{6M}\right)$.

(b) State and prove Schottky's theorem.

8. (a) If f is an entire function that omits two values, then show that f is a constant.

(b) State and prove Great Picard theorem.

9. (a) Define integral function. Show that the most general integral function with no zero is of the form $e^{g(z)}$, where $g(z)$ is itself an integral function.

(b) List the properties satisfied by Poisson Kernel.

(c) What do you mean by univalent functions ?

(d) Find the order of $\sin z$ and $\cos z$.

(e) Given that $\sin^2 z = 1$ holds for all complex values of z.