No. of Printed Pages : 02 **Roll No. ....**

DD311

M. Sc. EXAMINATION, 2020

(Fourth Semester)

(B Scheme) (Re-appear)

MATHEMATICS

MAT602B

Functional Analysis

Time : 3 *Hours*] [*Maximum Marks* : 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Every metric on a vector space can be obtained from a norm. Prove or disprove this statement.

(b) State and prove Minkowski inequality.

2. Prove that C[a, b] is a Banach space.

Unit II

3. (a) State and prove Hahn Banach extension theorem (Real form).

(b) State and prove uniform boundedness principle.

4. (a) State and prove Riesz representation theorem for bounded linear functional on L^p.

   (b) State and prove closed graph theorem.

Unit III

5. (a) Define a Compact Operator. Prove that a compact operator is bounded (Continuous) operator.

   (b) Let $\{T_n\}$ be a sequence of compact operators from a normed space N into a Banach space B. If $\|T_n - T\| \to 0$ as $n \to \infty$, then T is compact.

6. (a) State and prove Banach contraction principle.

   (b) State and prove Picard-Lindeloff theorem.

Unit IV

7. (a) State and prove Schwarz's inequality.

   (b) State and prove Projection theorem.

8. (a) State and prove Riesz representation theorem for continuous functionals on a Hilbert space.

   (b) Explain Gram-Schmidt orthogonalization process.