(ii) If dim X = ∞, X is a normed linear space, then identify operator is not compact.

(b) Let X and Y be normed spaces and $T : X \to Y$ be a linear operator. Then T is compact iff it maps every bounded sequence $<x_n>$ in X onto a sequence $<Tx_n>$ in Y which has a convergent subsequence. **10**

6. (a) Define fixed point. State and prove Banach contraction principle. **10**

(b) State and prove Picard's Theorem. **10**

Unit IV

7. (a) Define inner product space and Hilbert space. Further, state and prove Schwarz inequality in a Hilbert space H. **10**

(b) A closed convex subset C of a Hilbert space H contains a unique vector of smallest norm. **10**

DD311

M. Sc. EXAMINATION, May 2019

(Fourth Semester)

(B. Scheme) (Main & Re-appear)

MATHEMATICS

MAT602B

Functional Analysis

Time : 3 *Hours*] [*Maximum Marks :* 100

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit. All questions carry equal marks.

Unit I

1. (a) Define normed linear spaces. Prove that a normed linear space is a metric space with respect to metric d defined by $d(x, y) = \|x - y\|$ for all $x, y \in$ N. **10**

   (b) State and prove Holder's inequality for sequence. **10**

2. (a) If M is a closed linear subspace of a normed linear space N, then the quotient space N/M is a normed linear space with norm of each coset x + M defined by :

$$\|x + \text{M}\| = \inf\left\{\|x + \text{M}\| m \in \text{M}\right\}$$

   Further if N is a Banach space, then the quotient space N/M is also a Banach space with above defined norm. **10**

   (b) Let N and N' be normed linear spaces and let T be a bounded linear transformation of N into N'. Put :

$$a = \sup\left\{\|\text{T}x\| : x \in \text{N}, \|x\| = 1\right\}$$

$$b = \sup\left\{\frac{\|\text{T}x\|}{\|x\|} : x \in \text{N}, x \neq 0\right\}$$

$$c = \inf\left\{\text{K} : \text{K} \geq 0 : \|\text{T}x\| \leq k\|x\| \forall x \in \text{N}\right\}$$

   Then $\|\text{T}\| = \|a\| = b = c$. **10**

Unit II

3. (a) State and prove Hahn Banach theorem for reals. **16**

   (b) Define second conjugate and reflexive spaces. **4**

4. (a) State and prove open mapping theorem. **16**

   (b) (i) Define graph of f on a linear transformation from N to N', where N and N' are normed linear spaces.

   (ii) Define equivalent norm. **4**

Unit III

5. (a) Define compact operator and prove the following :

   (i) Every compact operator from a normed linear space X to normed linear space Y is continuous. **10**

8. (a) If M is a closed linear subspace of a Hilbert space H, then $H = M \oplus M^{\perp}$. **10**

(b) If $\{e_i\}$ is an orthogonal set in a Hilbert space H, then : **10**

$$\sum \left|\langle x, e_i \rangle\right| \le \|x\|^2 \ \forall\ x \in H$$

8. (a) If M is a closed linear subspace of a Hilbert space H, then $H = M \oplus M^{\perp}$. **10**

(b) If $\{e_i\}$ is an orthogonal set in a Hilbert space H, then : **10**

$$\sum \left|\langle x, e_i \rangle\right| \le \|x\|^2 \ \forall\ x \in H$$