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M.Sc. EXAMINATION, 2020

(Fourth Semester)

(C Scheme) (Re-appear)

MATHEMATICS

MAT602C

Inner Product Spaces and Advanced Measure Theory

Time : 2½ *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Four* questions in all. All questions carry equal marks.

1. (a) Prove that a closed convex subsets C of a Hilbert space H contains a unique vector of smallest norm.

(b) State and prove Bessel's inequality for countable orthonormal sets.

2. (a) State and prove Projection theorem.

(b) State and prove Riesz representation theorem in Hilbert spaces.

3. Prove that if H is a Hilbert spaces then it is reflexive.

4. (a) Prove that an operator T on a Hilbert space H is self adjoint iff (Tx, x) is real for all x.

(b) Prove that an operator T on a Hilbert space H is unitary iff it is an isometric isomorphism of H onto itself.

5. (a) Define signed measure and state and prove Hahn decomposition theorem.

   (b) State and prove Lebesgue decomposition theorem.

6. State and prove Radon–Nikodyn theorem.

7. (a) Discuss the following Lp spaces. State and prove Jensen's inequalities.

   (b) State and prove Caratheodory extension theorem.

8. (a) Discuss Bairemeasure, continuous functions with compact support and regularity of measure on locally compact spaces.

   (b) State and prove Riesz-Markoff theorem.

9. (i) State and prove Schwarz's inequality.

   (ii) Define orthogonal and orthonormal sets.

   (ii) Define adjoint and self adjoint of an operator.

   (iv) Define mutually signed measure.

   (v) Define Lebesgue-Stieltjes integral and Mutually signed measure.

   (vi) Define Product measure and State Fubini's theorem.