(b) By using integral test, test the behaviour of the series :

$$
\sum_{n=2}^{\infty} \frac{1}{n(\log n)^{p}} \cdot p>0
$$

## Unit IV

7. Define absolute convergence and conditional convergence for alternating series and test the convergence and absolute convergence of the series :

$$
\sum_{n=1}^{\infty}(-1)^{n-1} \sin \frac{1}{n}
$$

8. (a) Discuss the convergence of the series : 7

$$
\sum_{n=1}^{\infty} \frac{(\mathrm{L} n)^{2}}{\mathrm{~L}(2 n)} \cos n \theta
$$

(b) Show that the infinite product :

$$
\frac{1}{2} \cdot \frac{3}{2} \cdot \frac{3}{4} \cdot \frac{5}{4} \cdot \frac{5}{6} \ldots \ldots \ldots \cdot \frac{2 n-1}{2 n} \cdot \frac{2 n+1}{2 n}
$$

converges as $n \rightarrow \infty$.

Roll No. $\qquad$

## DD-341

## M. Sc. EXAMINATION, May 2017

(5 Years Integrated)
(Fourth Semester)
(Main \& Re-appear)
MATHEMATICS

## MAT-312-H

Sequences and Series

Time : 3 Hours]
[Maximum Marks : 75
Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt Five questions in all, selecting at least one question from each Unit. All questions carry equal marks.
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## Unit I

1. (a) Show that every non-empty subset of real number which is bounded below has a real number as its intimum.

7
(b) Prove that the interior of set A is the largest open subset of A.

8
2. (a) Write down the following definition with one examples :
(i) Limit point of a set
(ii) Derived set
(iii) Adherent pont
(iv) Closure of a set.
(b) Prove that every set satisfying the Heine Borel property is a compact set.

## Unit II

3. (a) State and prove Cauchy's second theorem on limits.

8
(b) Show that a sequence converges iff it is a Cauchy's sequence.
4. (a) Test the convergence of the following series :
(i) $\sum_{n=1}^{\infty}\left[\sqrt{n^{4}+1}-n^{2}\right]$
(ii) $\sum_{n=1}^{\infty} \cot ^{-1} n^{2}$
(b) Discuss the convergence of the series :

$$
\begin{equation*}
\sum_{n=1}^{\infty} \frac{x^{n}}{a^{n}+x^{n}} \cdot x>0 \tag{7}
\end{equation*}
$$

## Unit III

5. (a) Test the convergence of the series : 8 $\frac{1}{1.2 .3}+\frac{x}{4.5 .6}+\frac{x^{2}}{7.8 .9}+\ldots \ldots \ldots .(x>0)$
(b) Test the convergence of the series : 7

$$
\sum_{n=1}^{\infty} \frac{(1+n x)^{n}}{n^{n}}
$$

6. (a) Test the convergence of the series : $\mathbf{8}$

$$
1+\frac{x}{2}+\frac{\mathrm{L}^{2}}{3^{2}} x^{2}+\frac{\mathrm{L}^{3}}{4^{3}} x^{3}+\frac{\mathrm{L}^{4}}{5^{4}} x^{4}+\ldots \ldots \ldots
$$

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