Unit IV

7. (a) Show that the series $x - \frac{x^3}{3} + \frac{x^5}{5}$....... is absolutely convergent if $|x| < 1$ but conditionally convergent for $|x| = 1$. **8**

(b) State and prove Dirichlet's test. **7**

8. (a) Prove that the Cauchy product of the two series $3 + \sum_{n=1}^{\infty} 3^n$ and $-2 + \sum_{n=1}^{\infty} 2^n$ is absolutely convergent although both series are divergent. **8**

(b) Prove that $\sum_{n=1}^{\infty} \left(1 + \frac{x}{n}\right) e^{-x/n}$ is absolutely convergent for all real x. **7**

DD341

M.Sc. (5 Years Integrated) EXAMINATION, May 2019

(Fourth Semester)

(B. Scheme) (Re-appear)

B.Sc. (Hons.) M.Sc. (Mathematics)

MATHEMATICS

MAT312H

SEQUENCES AND SERIES

Time : 3 *Hours*] [*Maximum Marks* : 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Define L.U.B. and G.L.B. of a set. Prove that every non-empty subset of real numbers which is bounded below has a real number as its G.L.B. **8**

(b) Prove that the interior of a Set A is the largest open subset of A. **7**

2. (a) State and prove Bolzano-Weirstrass theorem. **8**

(b) Prove that every set satisfying the Heine Borel property is a compact set. **7**

Unit II

3. (a) Prove that every convergent sequence is bounded but not conversely. **8**

(b) Prove that $\lim\limits_{n\to\infty}\left(1+\frac{1}{n}\right)^n$ exists and lies between 2 and 3. **7**

4. (a) If the series $\sum\limits_{n=1}^{\infty} a_n$ converges, then $\lim\limits_{n\to\infty} a_n = 0$. Is the converse true ? Give example. **8**

(b) Test the convergence of the series whose nth term is $\frac{1}{\sqrt{n}}\sin\frac{1}{n}$. **7**

Unit III

5. (a) Discuss the convergence of the series : **8**

$$\frac{1}{2}+\left(\frac{2}{3}\right)x+\left(\frac{3}{4}\right)^2 x^2+\left(\frac{4}{5}\right)^3 x^3+........$$

(b) Test the convergence of the series : **7**

$$1+\frac{x}{2}+\frac{2!}{3^2}x^2+\frac{3!}{4^3}x^3+\frac{4!}{5^4}x^4$$
$$+............(x>0)$$

6. State and prove Cauchy's integral test. **15**