(b) Prove that a filter F on set X is an ultrafilter iff F contains all those subsets of X which intersects every member of F.

Unit IV

7. (a) Define covering mapping by giving an example and show that a covering map is a local homeomorphism.

(b) Define paracompactness by giving an example. Also prove that every paracompact Hausdorff space X is normal.

8. State and prove Nagata-Smirnov metrization theorem.

HH-343

M. Sc. EXAMINATION, May 2017

(5 Years Integrated)

(Eighth Semester)

(Main & Re-appear)

MATHEMATICS

MAT-516-H

General Topology

Time : 3 *Hours*] [*Maximum Marks :* 75

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Unit.

Unit I

1. (a) Prove that every homeomorphic image of a completely normal space is completely normal.

   (b) Define T_4-space by giving an example. Give an example of a topological space which is normal but not a T_4-space. **5**

   (c) Show that the property of being a completely regular space is hereditary.

2. (a) If f is a continuous mapping of the topological space (X, T) onto the space (Y, U) such that f is either open or closed. Then prove that U is the quotient topology for Y. **5**

   (b) If the open set G has a non-empty intersection with a connected set C in a T_4-space X, then either C consists of one point or the set $C \cap G$ has cardinality greater than or equal to the cardinality of reals. **10**

Unit II

3. (a) Prove that the product space $X = P(X_i : i \in I)$ is a T_1-space iff each coordinate space is T_1-space.

   (b) Define projection mapping of a product topological space. Also prove that each projection $\pi_j : X \to X_j$ of a product space $X = \prod_i X_i$ is an open mapping.

4. State and prove Embedding lemma.

Unit III

5. (a) A topological space (X, T) is Hausdorff iff every net in X can converge to atmost one point.

   (b) Let F be a filter on a non-empty set X and let $A \subset X$. Then show that $\exists$ a filter **F′** finer than F such that $A \in$ **F′** iff $A \cap B \neq \phi$ for every $B \in F$.

6. (a) Prove that every filter F on set X is the intersection of all the ultrafilters finer than F.