(b) Let $\{f_n\}$ be a sequence of measurable function which converge to f a.e. on E with $m(E) < \infty$.

Then $f_n \xrightarrow{m} f$ on E

Show that the converse of above is not true. **7**

Part III

5. (a) Define Reimann integral and Lebesgue integral. Point out the shortcomings of Reimann integration. **8**

(b) A bounded function f defined on a measurable set E of finite measure is Lebesgue integrable. Show that f is measurable. **7**

6. (a) Let f and g be bounded measurable functions defined on a set E of finite measure. Then. **8**

(i) $\int_E af = a\int_E f$, for all real numbers a.

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HH-341

Dual Degree B.Sc. (Hons.)(Mathematics)/ M.Sc. (Mathematics) EXAMINATION, May 2018

(Eighth Semester)

(Main & Re-appear)

MAT512H

MEASURE AND INTEGRATION

Time : 3 Hours] [*Maximum Marks : 75*

Before answering the question-paper candidates should ensure that they have been supplied to correct and complete question-paper. No complaint, in this regard, will be entertained after the examination.

Note : Attempt *Five* questions in all, selecting at least *one* question from each Part.

Part I

1. (a) Prove that outer measure of an interval is its length. **8**

(b) Let $E_1, E_2, \ldots\ldots, E_n$ be a finite sequence of disjoint measurable sets. Then for any set A, **7**

$$m^*\left(A \cap \left[\bigcup_{i=1}^{n} E_i\right]\right) = \sum_{i=1}^{n} m^*(A \cap E_i)$$

2. (a) Let E_i be an infinite increasing sequence of sets (not necessarily measurable) then : **8**

$$m^*\left(\bigcup_{i=1}^{\infty} E_i\right) = \lim_{n\to\infty} m^*(E_n)$$

(b) Show that there exists a non-measurable set in the interval [0, 1]. **7**

Part II

3. (a) Define a measurable function. Let f be a function defined on a measurable set E. Then f is measurable if and only if, for any open set G in R, the inverse image $f^{-1}(G)$ is a measurable set. **8**

(b) Prove that a continuous function defined on a measurable set is measurable. Show that the converse may not be true. **7**

4. (a) Let E be a measurable set with $m(E) < \infty$, and $\{f_n\}$ a sequence of measurable functions defined on E. Let f be a measurable real valued function such that $f_n(x) \to f(x)$ for each $x \in E$. Then given $\in < 0$ and $\delta > 0$, $\exists$ a measurable set $A \subset E$ with $m(A) < \delta$ and an integer N such that :

$|f_n(x) \to f(x) < \in|$, for all $x \in E - A$ and all $n \geq N$. **8**

8. (a) If $f \in R[a, b]$ and α is a monotonic increasing on $[a, b]$ such that $\alpha' \in R[a, b]$, then $f \in R(\alpha)$ and : **8**

$$\int_a^\mu f d\alpha = \int_a^\mu f \alpha' dx$$

(b) Solve : **7**

(i) $\int_0^2 [x] dx^2$

(ii) $\int_0^{\pi/2} x d(\sin x)$.

(ii) $\int_E (f+g) = \int_E f + \int_E g$

(iii) If $f = g$ a.e., then $\int_E f = \int_E g$

Is the conversed true ?

(b) Let f be a non-negative function which is integrable over a set E. Then given $\in > 0$, there is a $\delta > 0$ such that for every set $A \subset E$ with $m(A) < \delta$, we have : **7**

$$\int_A f < \in$$

Part IV

7. (a) A function f is integrable with respect to α on $[a, b]$ if and only if for every $\in > 0$ there exists a partition P of $[a, b]$ such that : **8**

$$U(p, f, \alpha) - L(p, f, \alpha) < \in$$

(b) If $f \in R(\alpha_1)$ and $\int_\mu R(\alpha_2)$, then $f \in R(\alpha_1 + \alpha_2)$ and : **7**

$$\int_a^\mu f d(\alpha_1 + \alpha_2) = \int_a^\mu f d\alpha_1 + \int_a^\mu f d\alpha_1$$